Course: Company Finance Management

Field of study: Finance and Accounting

Form of classes and number of hours: lecture, practice, and project 75 h

Number of ECTS credits: 6

Learning outcomes:

Knowledge:

- Student knows about the financing strategy for business assets.
- Student knows about the sources of capital of a company and the valuation of securities.
- Student knows the methods of estimating financial risk.
- Student can interpret the relationship between the expected rate of return on capital investment as well as the cost of capital and risk.
- Student knows the Capital Asset Pricing Model (CAPM), Capital Market Line (CML), Security Market Line (SML), Characteristic Line of an asset.
- Student understands the sense and methods of calculating the cost of equity, debt as well as the weighted average cost of capital.

Skills:

- Student can characterise the sources of financing business activities and assess the selection of appropriate sources of financing.
- Student can perform the valuation of securities: stocks, bonds.
- Student can plot the relationship between the expected rate of return and risk of a portfolio, and thus estimate the cost of capital.
- Student can assess cost of equity using the CAPM.

Social competences:

- Student understands the meaning of financial decisions in a company.
- Student recognises the need of taking into account as well as appropriate assessment of risk and uncertainty in business.
- Student understands the goals of business and the fact that in a competitive economy a capital investment should bring financial benefits.
- Student understands the sense of cooperation in a group.

Evaluation methods of learning outcomes:

exam, test, observation during project implementation, project evaluation

List of course topics:

Lecture:

1. Introduction: objectives and the role of financial management.
2. Capital and money market as a source of financing.
3. Debt capital sources of financing and their cost.
4. Equity capital sources of financing and their cost.
5. Time value of money.
6. Investment risk and rate of return.
7. Methods of securities valuation (price and required rate of return).

8. Risk and methods of measurement of investment activity risk.
9. Statistical risk measures.
10. Portfolio theory and the diversification of risk.
11. Market cost of risk (CML).
12. Cost of equity – CAPM method.
13. Weighted average cost of capital.
14. The impact of the financial structure on the value of a company. MM models.
15. The impact of the financial structure on the value of a company. MM models.

Practical classes:

1. Future value of money.
2. Present value of money.
3. Credit amortisation.
4. Valuation of securities – stocks.
5. Estimating the number of shares and their issue price.
6. Valuation of securities – bonds.
7. Statistical risk measures.
8. Portfolio theory – two-element portfolio.
9. Market cost of risk (CML).
10. Beta coefficient calculation.
11. Cost of equity (CAPM).
12. Weighted average cost of capital.
13. Market index model.
14. WACC.
15. Cost of leasing.

Project:

1. Financial structure optimisation.
   - Project preparation,
   - Scenario analysis,
   - Scenario evaluation through financial ratios,
   - Preparation of results and their interpretation and presentation.
2. Calculation of the number of shares and their issue price for a selected company.

Bibliography

[1] Brealey R. A., Myers S. C., *Principles of Corporate Finance*. The McGraw-Hill Companies, Inc., Printed in the United States of America, 1996.

[2] Brigham E. F., Gapensky L. C., *Financial Management*. The Dryden Press, USA 1988.

[3] Brigham E. F., Ehrhardt M. C., *Financial Management: Theory and Practice*. 11 ed. – Mason, Thomson, South-Western Cengage Learning, 2005.

[4] McGuigan J. R., Kretlow W. J., Moyer R. Ch., *Contemporary Corporate Finance.* 11 ed., internat. student ed. – Mason, South-Western Cengage Learning, Thomson, 2009.

[5] Moyer R. Ch., McGuigan J. R., Kretlow W. J., *Contemporary Corporate Finance*. Wes Publishing Company, USA 1987.

[6] Wilimowska Z., Szczepańska J., *Selected Aspects of Economy in Industrial Practice*. Politechnika Warszawska, Warszawa 2005.

Table of contents

1. Introduction

The basis of a well-implemented corporate financial management is true, reliable, objective, well-prepared and easily accessible information about a company and its financial environment.

This information, in accordance with the art of management, is meant to allow for:

- assessing the current financial condition of a company in a reliable manner,
- assessing in a reliable manner the following possibilities:
  - the development of a company in the next period (e.g. a year),
  - the development of a company in the longer period (e.g. 5 years),
  - the strategic development of a company (e.g. 10-20 years),
- setting strategic goals,
- designing a way to reach a strategic goal through the points designated by short- and medium-term planning,
- assessing risk related to the way to reach the strategic goal,
- analysing risk and breaking it down into elementary risks, determining the causes of each of these partial risks,
- developing ways to eliminate some elementary risks and ways to protect against elementary risks that cannot be eliminated.

Company finance management means

- Sources of financing – capital structure creation
- Investment – asset structure creation

Taking into consideration a risk which is associated with these activity.

A firm consists of not only tangible and intangible assets, but should be considered together with the sources of financing of said assets (Fig. 1). Its current and basic business activity needs financing:

- Enterprise creation (start up),
- Rationalisation of the business activity,
- Development,
- Company recovery,
- etc.

An enterprise constitutes not only material infrastructure. In the modern economy, it should be considered together with the owners of capital financing the operations of a company. Hence, in managing the finances of an enterprise, attention should be paid primarily to those stakeholders who invest their savings in the business conducted by an enterprise (Fig. 1.) and who expect to gain appropriate financial benefits in return for the invested funds.

For lenders, financial benefits are, of course, interest, while for owners of equity, expected benefits are primarily the dividend paid by the company and capital growth understood as an increase in value, for example, in the price of shares or stocks.

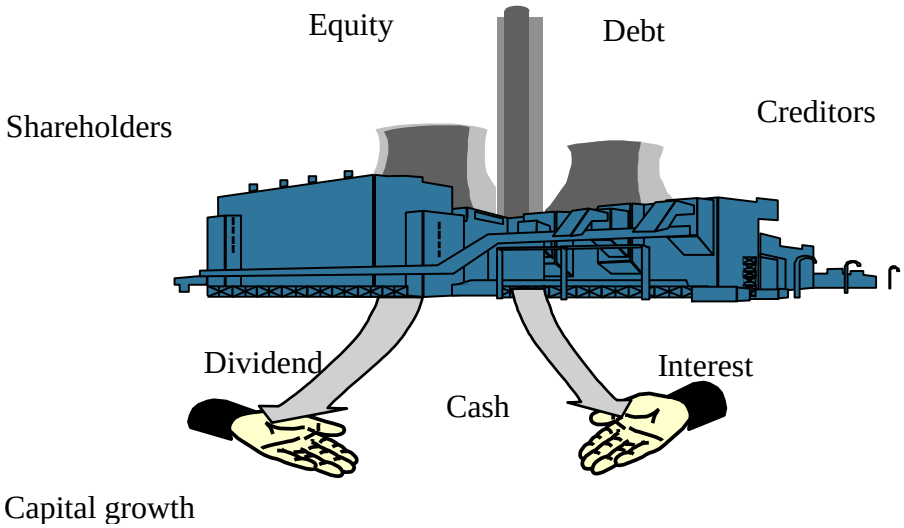


Figure 1. A firm and its capital owners

2. Financing

2.1. Financial market

In modern economies, enterprises look for sources of financing in financial markets (Fig. 2), especially long-term in nature.

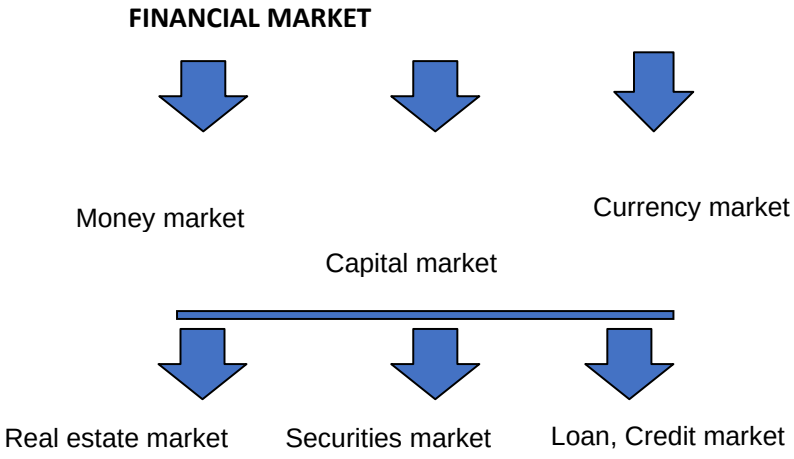


Figure 2. Financial market structure

In financial market, there are two sides participating, two investor types for the sale-purchase transaction concerning a capital. One who invest its financial surpluses and a different one who needs the capital for the development of an investment (Fig. 3).

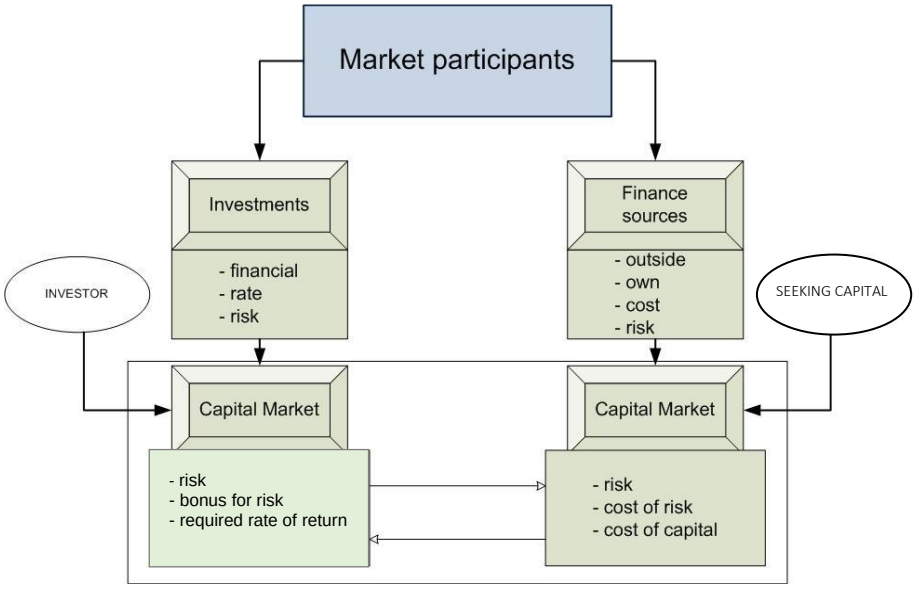


Figure 3. Transaction at financial market

An investment is a capital cost incurred today for expected benefits in the future. The current moment is known, but the future is secret.

It means that an investment is a renunciation of what is certain for uncertain future benefits

So both sides of transaction have to consider many factors (Fig. 4).

Investor **Capital seller**	**Investor** **Capital buyer**
Risk of investment	**Risk of using one's capital**
Risk premium	**Cost of risk**
Rate of return	**Cost of capital**

Figure 4. Transaction factors

For the sale of capital on the financial market to be successful, both sides of a transaction, an investor and a company that buys one's capital on the market must obtain a certain consensus about the price of said capital.

What is considered to be earnings for one side, for the other means making the cost of obtaining capital.

Therefore, both sides of the purchase-sale transaction concerning a capital should always be considered in the financial market, in particular due to the risk of, on the one hand, investment, and on the other hand, due to the risk of using said capital.

A transaction risk can be understood as:

- the possibility of loss,
- the probability of loss,
- the discrepancy between the actual and expected results,
- the probability of a different result than expected,

- the potential, possibility of adverse events,
- the conditions in which there is a possibility of loss,
- the uncertainty, risk, the possibility of failure to execute an order,
- the possibility of not achieving expected results established at the time of decision-making.

2.2. Sources of financing

There are many internal and external sources of financing (Fig. 5 and 6).

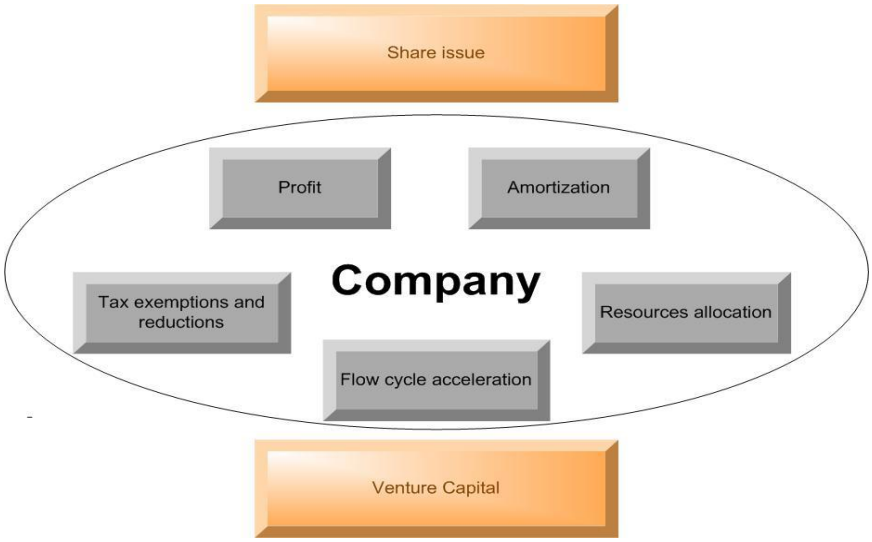


Figure 5. External and internal equity capital

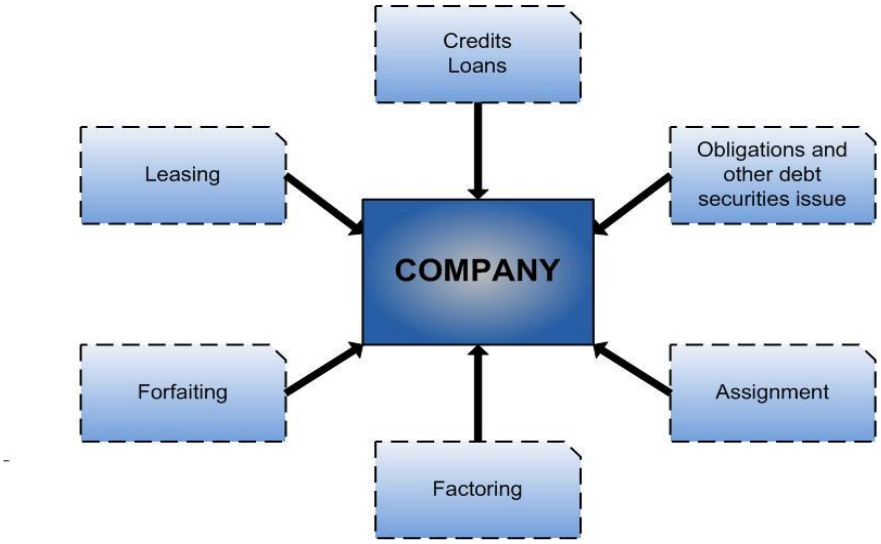


Figure 6. External and internal debt capital

In order to choose the right source, a company requires selection criteria (Fig. 7).

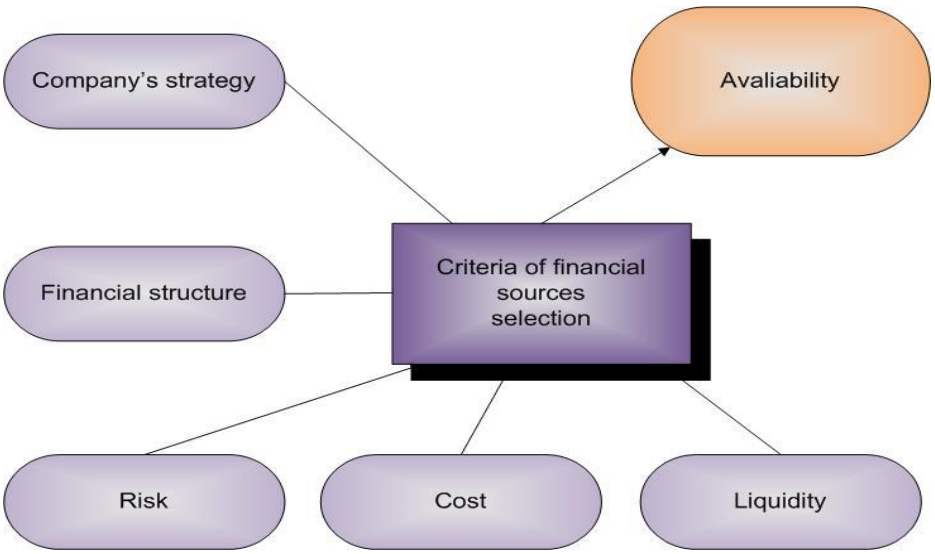


Figure 7. Criteria of financial sources selection

Of course, all criteria are important and should be taken into consideration when choosing financing sources. But their risk and cost constitute such criteria which require special tools to evaluate the right decisions.

3. Risk and its measurement

3.1. Concept of risk

Risk (it. *Risico*) is mainly assumed to be an undertaking, the result of which is unknown or uncertain, or the possibility that something may turn out well or not, or future alternatives if possibilities are known.

The concept of risk is a complex one, it has a lot of aspects.

There are various definitions in different literature sources.

The act of purchasing and selling a capital is accomplished on the capital market. Both participants of the transaction: capital seller (investor) and purchaser (acquirer), consider transaction profitability from the perspective of risk.

Types of risk depend on parameters, which determine threats for the objective of the investment in capital assets:

- political risk;
- risk connected with the situation on stock exchange;
- market risk;
- risk of liquidity;
- inflation risk;
- risk of rate of interest;
- risk of currency exchange;
- risk of failure to meet the conditions;
- risk of reinvestment;
- risk of redemption request;
- risk of convertibility;
- financial risk.

3.2. Selected statistical measures of risk

It is assumed that the rate of return *r* for an investor is a random variable.

Probability

In the moment of decision making, one can only try to define the probability of an event occurrence in which the realisation of a random *r*, *R*, should be higher than *r** – the expected rate of return (*R>r**);

P(R≥r)*, where *P* is the probability.

If an opposite situation occurred (an event is unfavourable), the probability of such event is a risk *s.*

Thus:

$$s = P(R < r^*) = F_R(r^*) = \int_{-\infty}^{r^*} f_R(r)dr$$

$$P(R < \infty^*) = 1,\ P(R > \infty^*) = 0$$

Expected rate of return

For the real type of the rate of return

$$\overline{R} = \int_{-\infty}^{\infty} r(R)RdR$$

$\overline{R}$ – expected rate of return

f(R) – probability density distribution for the rate of return *R*

For the discrete type of the rate of return:

$$\overline{R} = \sum_{i=1}^{n} p_i * r_i$$

p_i – probability of *i*-value occurrence
r_i – value of *i*-rate of return

The less the expected value of *r* is, the greater the risk.

Standard deviation

For the real type of the rate of return:

$$\sigma = \sqrt{\int_{-\infty}^{\infty} f(R)(R-\overline{R})^2 dR}$$

For the discrete type of the rate of return:

$$\sigma = \sqrt{\sum_{i=1}^{n} (r_i - \overline{R})^2 * p_i}$$

The less the value of standard deviation is, the lesser the risk, because the value of deviation shows how big is the value dispersion of the rate of return in the vicinity of the expected value (Fig. 8).

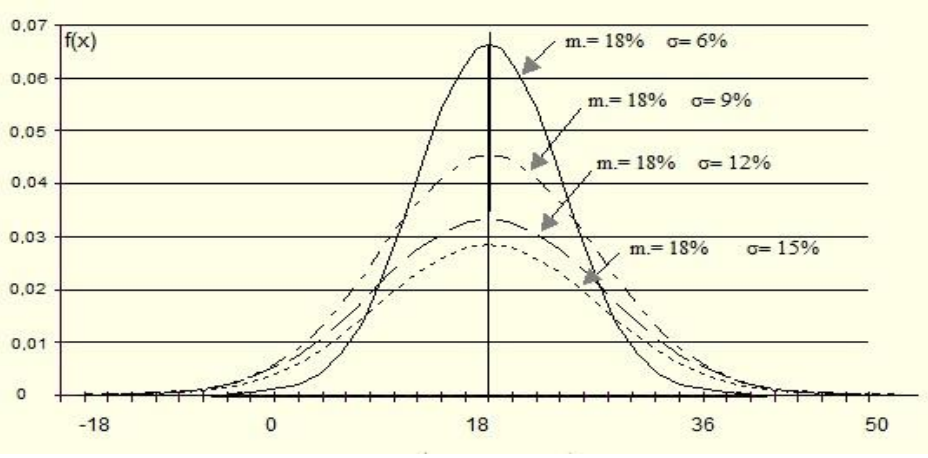


Figure 8. Probability density distribution functions

with expected value = 18% and σ = 15%, 12%, 9%, 6%

Relative risk measurement

If alternative investments have the same expected value of the return rate, then the choice of an investor poses no problem. If not, it means that the expected values for two investments differ (Fig. 9).

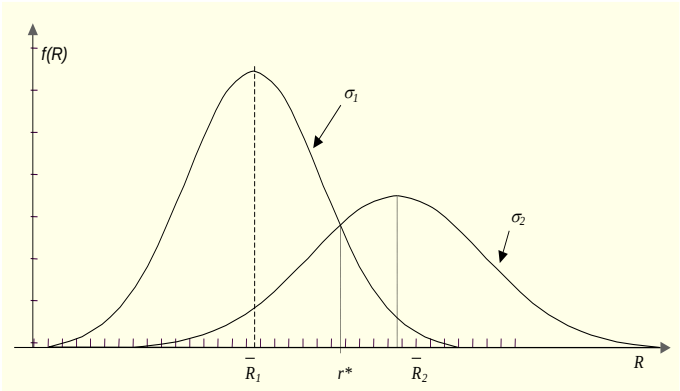


Figure 9. Probability density distribution functions with different expected values and different standard deviations

In this case, it is necessary to use the relative risk measure, e.g. the coefficient of variation:

$$v = \frac{\sigma}{\overline{R}}$$

4. Risk pricing

The portfolio theory is applied to the price risk on the capital market defined by Markowitz who is known as the 'father of the portfolio theory'.

In such a case, it can be observed that if an investor builds a set of capital investments, the risk can be lower than the weighted average of risk for capital assets included in the portfolio.

Let us assume that there is a portfolio that is comprised of two elements: A and B, in the case of which the expected values of rates of return for A and B and standard deviations of rates of return

for A and B are known. In the case of said portfolio, where the share of the security *A* is W_A, and security *B* is W_B, assuming that $W_A+W_B = 1$, the formula is as follows:

$$\overline{R}_p = W_A\overline{R}_A + W_B\overline{R}_B$$

Risk of portfolio measure by the standard deviation is calculated via the following formula:

$$\sigma_{AB} = \sigma_p = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_B\rho_{AB}\sigma_A\sigma_B}$$

σ_{AB}, σ_p – standard deviation of the rate of return, portfolio risk,

σ^2_A, σ^2_B – variation of the rate of return from securities *A* and *B*,

σ_A, σ_B – standard deviation of the rate of return from the investment in securities A and B,

ρ_{AB} – correlation index between the rates of return from securities A and B.

All portfolios depending on correlation indexes of the rate of returns and proportion of investing capital in *A* or *B* capital assets are presented in Figure 10.

As you can observe, it is possible, in the extreme case, to create risk a free portfolio.

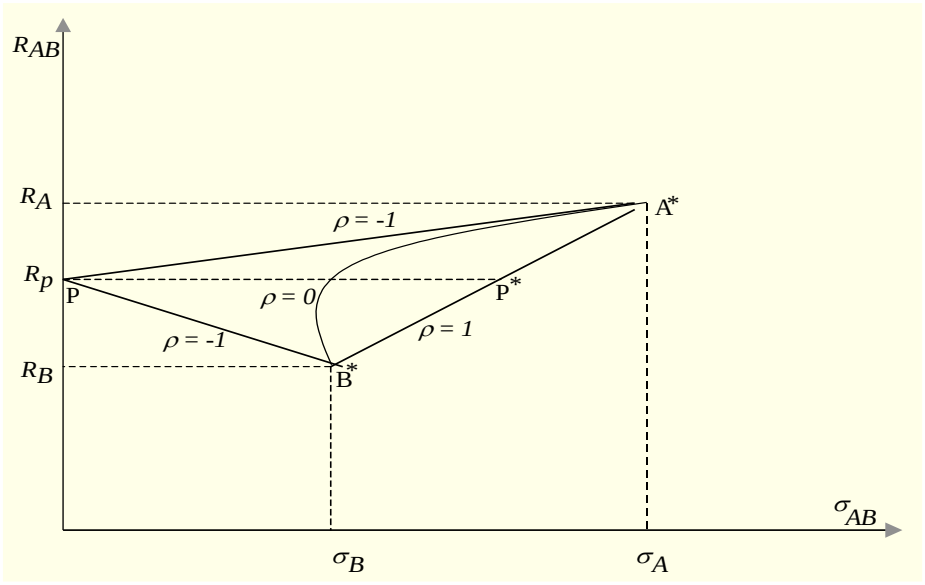


Figure 10. Relation between the rate of return and the portfolio risk

4.1. Capital Market Line CML

Let us consider a portfolio comprised of two elements (X_1, X_2), where X_1 – a risky asset and X_2 – a risk-free asset.

Then:

$$\overline{R}_p = w_1\overline{R}_1 + w_2\overline{R}_2 = w_1\overline{R}_1 + w_2R_f \text{ ,}$$

R_f – risk-free rate of return
w_1 – proportion of the invested capital in X_1,
w_2 – proportion of the invested capital in X_2.

Portfolio risk:

$$\sigma_p = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2} .$$

Because $\sigma_2 = 0$, so:

$$\sigma_p = \sqrt{\sigma_1^2 w_1^2} = \sigma_1 w_1,$$

Then:

$$w_1 = \frac{\sigma_p}{\sigma_1} .$$

Next:

$$\begin{aligned} \overline{R}_p &= w_1 \overline{R}_1 + w_2 R_f = \\ &= \frac{\sigma_p}{\sigma_1} \overline{R}_1 + \left(1 - \frac{\sigma_p}{\sigma_1}\right) R_f = \\ &= R_f + \left(\overline{R}_1 - R_f\right) \frac{\sigma_p}{\sigma_1} = \\ &= R_f + \frac{\overline{R}_1 - R_f}{\sigma_1} \sigma_p \end{aligned}$$

So, as you can see, that dependency $\overline{R}_P$ = f (σ_P) is a linear function.

If the asset X_1 is the market asset, it means that this is a set of all n assets that existed at capital market and *Wi* proportion of capital is invested in each asset, where:

$$w_i = \frac{market\ value\ of\ asset\ i}{market\ value\ of\ all\ assets}$$

i = 1, 2, …, n

The linear function is called the CML,

$$\overline{R}_p = R_f + \left(\frac{\overline{R}_m - R_f}{\sigma_m}\right) \sigma_p$$

R_f – risk-free rate,
$\overline{R}_m$ – expected rate of the market portfolio – market rate of return,
σ_m – standard deviation of the market portfolio,
$\overline{R}_p$ – expected return from portfolio located on the CML,
σ_p – standard deviation of the portfolio.

The slope coefficient of this line is the market price of risk.

Market price of risk = $\frac{\overline{R}_m - R_f}{\sigma_m}$

4.2. CAPM – Capital Asset Pricing Model

Let's create a portfolio consisting of the *i-th* asset (risky asset) and the market asset so that it is an effective one. Let us assume that we have invested the *w-th* part of our funds in one asset *i-th* and in the market portfolio *(1-w)th*.

Then:

$$\overline{R}_p = w\overline{R}_i + (1-w)\overline{R}_m$$

and

$$\sigma_p^2 = w^2\sigma_i^2 + (1-w)^2\sigma_m^2 + 2(1-w)w\sigma_{im}$$

After some transformations, we obtain the following:

$$\overline{R}_i = R_f + (\overline{R}_m - R_f)\frac{\sigma_{im}}{\sigma_m^2}$$

This model is called the Capital Asset Pricing Model (CAPM) which allows us to value each capital asset on the basis of market parameters.

This model can also be written as:

$$\overline{R_i} = R_f + \left(\frac{\overline{R}_m - R_f}{\sigma_m}\right)\left(\frac{\sigma_{im}}{\sigma_m}\right)$$

which is a similar notation to the CML model or the most popular notation, the most known formula:

$$\overline{R_i} = R_f + (\overline{R}_m - R_f)\beta_i$$

where:

$\sigma_{im} = \sum_{j=1}^{N} w_j\sigma_{ij}$ – covariance of the return of the i- security with the return of the market

$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$ – beta risk of the i- security (relative risk (to the market)).

Example:
Suppose that the risk-free rate is 9% and the market rate is 15%.

Then:

$$\bar{R}_i = 0.09 + (0.15 - 0.09)\beta_i = 0.09 + 0.06\beta_i$$

whereas the risk premium is:

$$\bar{R}_m - R_f = 6\%$$

1. For the securities of $\beta i = 1$, that is, for the securities of the same beta as the market beta, the required rate of return will be the same as the market rate.

In this case:

$$R_i = 15\%.$$

2. For securities with a beta higher than the market beta, $\beta i > \beta m = 1$, the required rate of return is higher than the market rate. The price is growing faster than the market in the 'bull' phase (market growth) and decreases faster than the market in the 'bear' phase (decline in the market). These assets are called aggressive securities. They have a higher systematic risk and a greater required rate of return.

In the example, for $\beta i = 1.5$, the required rate of return is:

$$\bar{R}_i = 0.09 + 0.06 \cdot 1.5 = 18\%$$

It is 3% higher than the market rate.

3. For securities with a beta less than the market beta (e.g. $\beta i = 0.5$) = $\beta i < \beta m = 1$, the required rate of return is lower than the market. Price increases more slowly than the market in the 'bull' phase and falls slower in the 'bear' phase. In the case of this property, the shares are referred to as defensive shares – they have a less systematic risk and therefore, the required rate of return is lower than the market return.

In this case:

$$\bar{R}_i = 0.09 + 0.06 \cdot 0.5 = 12\%$$

5. Pricing of securities

The value of securities can be estimated as the present value of the future cash flows.

5.1. Share pricing

Future cash flows of shares for an investor are the dividends that are paid, so the price of shares can be estimated as the present value of the future dividends.

The expected cash flows from shares consist of two components:
- the expected annual dividend
- the price at which an investor expects to sell his share

Let us assume that the investment is for one year and that:
D_0 – dividend paid in the current year,
D_1 – dividend paid in the following year,
P_0 – current market price of a share,
P_1 – market price of a share at which an investor sells his share after one year,
k_s – required rate of return (taking into consideration the risk of investment).

Then:

$$k_s = \frac{D_1 + (P_1 - P_0)}{P_0} = \frac{D_1}{P_0} + \frac{(P_1 - P_0)}{P_0}$$

For one year investment the price of share is:

$$Po = \frac{D_1}{(1+k_s)} + \frac{P_1}{(1+k_s)}$$

And for n years of the investment:

$$P_0 = \frac{D_1}{(1+k_s)} + \frac{D_2}{(1+k_s)^2} + ... + \frac{D_n}{(1+k_s)^n} + \frac{P_n}{(1+k_s)^n} =$$

$$= \sum_{t=1}^{n} \frac{D_t}{(1+k_s)^t} + \frac{P_n}{(1+k_s)^n}$$

Prolonging the investment to infinity:

$$P_0 = \sum_{t=1}^{\infty} D_t (1+k_s)^{-t} .$$

It is impossible to calculate the sum of a sequence without establishing special restrictions on the elements of the sequence. The sum exists only for the convergent geometric sequence.

For example: if we assume that the dividend is constant over a period of time $D_1 = D_2 = ... = D$, and that ks is constant over time:

$$P_0 = \sum_{t=1}^{\infty} D_t (1+k_s)^{-t} = \sum_{t=1}^{\infty} D(1+k_s)^{-t} =$$
$$= D\sum_{t=1}^{\infty} (1+k_s)^{-t} = \frac{D}{k_s}$$

In order to obtain a stable policy for dividend payments and a constant growth *g* of net profit *ks >g*, the following applies:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_0(1+g)^t}{(1+k_s)^t} = D_0 \frac{(1+g)}{k_s - g} = \frac{D_1}{k_s - g}$$

This is known as the Gordon's model.

5.2. Bond price estimation

In the case of a fixed-rate bond, the bondholder receives over a certain pre-determined period of time (e.g. month, quarter of year, or one year) an interest at a fixed level, and the last interest payment at the redemption date plus nominal value.

$$C = \sum_{i=1}^{n} \frac{O}{(1+R)^i} + \frac{W_n}{(1+R)^n}$$

where:
C – bond price
O – interest
R – internal rate of the return of a bond
W_n – nominal value of a bond
n – number of years.

5.3. Example of share pricing

The managers of the company TOR S.A. are considering an investment project, the implementation of which requires PLN 10 000 000. After performing an analysis of the possible sources of finance, they have decided to issue new shares.

The board of directors ordered the financial department to determine the issue price of shares and the number of newly issued shares.

Step I.
The financial manager's first step was to make some arrangements.

First, the cost of issuing shares is *kc = 8%* of the gross proceeds.

Second, the company currently has 4 000 000 outstanding shares.

Third, the shareholders expect the rate of return to be $k_s = 12\%$.

Fourth, in accordance with the dividend policy of the company, the value of dividends paid should be *50%* of net profit.

Fifth, *g* – the expected growth rate of the company equals 3%.

Sixth, the expected profit from current operations for the next year is PLN 4 000 000.

Seventh, the expected rate of return on the invested capital should be *R = 15%*.

Eighth, to sell new shares in a timely manner, *Pd* of their price should be set at 80% of the market share of *P*.

Step II.
Considering the above findings, the manager's second step was to make calculations.

1. The inflow of capital from the issue will be as follows:

Ve = Vc - Vc = kc · PLN 10 000 000 (1 - 0.08) = PLN 9 200 000,

Where:
Ve – inflow of capital resulting from issuing,
Vc – required investment capital.

2. Profit from new investment

15% of PLN 9 200 000 = PLN 1 380 000

3. Total profit next year *zc*

zc = profit from continuing operations + return on investment =

= PLN 4 000 000 + PLN 1 380 000 = PLN 5 380 000

4. The value of next year's dividend

50% · zc = 0.5 · PLN 5 380 000 = PLN 2 690 000.

5. The market value of the company – the MV at the assumed growth of *g = 3%* and the required rate of return on shares *ks = 12%*

MV = D1 / (ks - g) = PLN 2 690 000 / (0.12 - 0.03) = PLN 2 690 000 /0.09 = PLN 29 888 888.

6. Denoting by *Lx* the number of newly issued shares, the number of shares after they are issued will be:

4 000 000 + Lx.

The market share at a price of P means that the total market value of the company must satisfy the result of the following equation:

(4 000 000 + Lx) · P = PLN 29 888 888.

7. The discounted price of the new shares is *Pd = 0.8 · P*, hence:

4 000 000 · Pd + Pd · Lx = PLN 23 911 110.

8. The required investment capital amounts to PLN 10 000 000, i.e.

Lx · Pd = PLN 10 000 000.

9. Considering points 7 and 8, the following is obtained:

4 000 000 · Pd + PLN 10 000 000 = PLN 23 911 110.

Therefore:

Pd = PLN 13 911 110 /4 000 000 = PLN 3.48.

10. The number of newly issued shares is calculated using the following equation:

Lx · Pd = PLN 10 000 000, the Lx = PLN 10 000 000 / PLN 3.48 = 2 873 563

Conclusion

First. The company shall issue 2 873 563 shares at the issue price of PLN 3.48 in order to achieve an increase in equity to PLN 10 000 000.

Second. The issue price per share of PLN 3.48 is discounted by 20% in comparison to the market price, which is:

P = PLN 3.48/0.08 = PLN 4.35.

Third. After the shares are issued, the company will have 6 873 563 issued shares with a total value of PLN 29 899 999.

6. Practical classes

6.1. Time value of money

A series of expenditures incurred in equal time intervals is called **cash outflow**, whereas a series of money proceeds is called **cash inflow**.

Simple interest – paid or earned interest only from the base capital.

	Fixed interest	Variable interest
Future value	$FV_n = PV + PV * n * r$	$FV_n = PV + PV * \sum_{i=1}^{n} r_i$
Present value	$PV = \frac{FV_n}{1 + n * r}$	$PV = \frac{FV_n}{1 + \sum_{i=1}^{n} r_i}$

Compound interest – paid or earned interest not only from the base capital, but also paid or earned interest in the previous billing period.

Compounding – a process of adding interest to the amount of capital – interest is accumulated with the amount of capital.

Compounding period – a period of time, after which interest is credited to the amount of capital.

	Fixed interest	Variable interest
Future value	$FV_n = PV(1+r)^n$ or $FV_n = PV * MWP_n^r$	$FV_n = PV\prod_{i=1}^{n}(1+r_i)$
Present value	$PV = \frac{FV_n}{(1+r)^n}$ or $PV = FV_n * MWP_n^r$	$PV = FV_n\left(\prod_{i=1}^{n}(1+r_i)\right)^{-1}$

Effective annual rate – a rate that would give the same interest rate in the case of one, annual capitalisation, as in the case in which we have *m* capitalisations during one year. Instead of a more frequent then annual capitalisation, one can use the effective rate and the annual capitalisation.

$$r_{ef} = \left(1+\frac{r_{nom}}{m}\right)^m - 1 \qquad FV_{m*n} = PV\left(1+\frac{r_{nom}}{m}\right)^{n*m}$$

Task 1. The initial capital amounts to PLN 2 000, and an applicable interest rate is 20%. Calculate the future capital value after 5 years if interest is calculated:

a) annually,
b) every six months.

Task 2. You have PLN 5 000. How long should you keep it in your bank to double this amount if the applicable interest rate is 8%.

Task 3. The AGROMA company was valued at PLN 100 000. An offer was made to the owner, according to which the company would be purchased at a price of PLN 180 000 and the date of payment would be by the end of the three-year period. The interest rate is estimated to be 20%. Is the offer good for the owner?

Task 4. The AGROMA company was valued at PLN 100 000. An offer was made to the owner, according to which the company would be purchased at a price of PLN 180 000 and the date of payment would be at the end of the three-year period. In the first year, the expected rate would be 25%, in the second 20%, and in the third 15%. Is this offer beneficial?

Annuity – a series of identical contributions or disbursements made at regular, definite time intervals, in which case the same interest rate is maintained. The value of the contributions or disbursements is referred to as the size of cash flow or payment and denoted with *PMT* (payment).

Annuity paid from the bottom (ordinary annuity) – payments occur at the end of every period of time.

Annuity paid in advance (annuity due) – payments occur at the beginning of every period of time.

Future Value of Annuity FV A (for the annuity due FV AD) – the amount of capital at the end of the last time period, that it is equivalent to the series of payments at the interest rate and the number of periods.

Present Value of Annuity PV A (for the annuity due PV AD) – the amount of capital at the time *t=0*, that is equivalent to the series of payments at the interest rate and the number of periods.

	Future Value of Annuity	Present Value of Annuity
Future value	$FV\ A = PMT\frac{(1+r)^n - 1}{r}$ or $FV\ A = PMT * MWPR_n^r$	$FV\ AD = PMT\frac{(1+r)^n - 1}{r}(1+r)$ or $FV\ AD = PMT * MWPR_n^r * (1+r)$
Present value	$PV\ A = PMT\frac{1-\frac{1}{(1+r)^n}}{r}$ or $PV\ A = PMT * MWOR_n^r$	$PV\ AD = PMT\frac{1-\frac{1}{(1+r)^n}}{r}(1+r)$ or $PV\ AD = PMT * MWOR_n^r * (1+r)$

Perpetuity – (normal) annuity with payments that will never end, meaning that the amount of payments is infinite. One can only calculate its present value, and the future value cannot be determined because normal annuity does not have any limit for $n \to \infty$.

$$PV_{perpetuity} = \frac{PMT}{r}$$

Balance between contributions and payments.

$$FV\ A_{n,r} = PV\ A_{n,r}$$

because:

$$FV\ A_{n,r} = PMT * MWPR_n^r \quad \text{and} \quad PV\ A_{n,r} = EMT * MWOR_n^r$$

that is why:

$$PMT * MWPR_n^r = EMT * MWOR_n^r$$

Task 1. The COMP company was valued at PLN 200 000. An offer was made to the owner, according to which the company would be purchased: payments would be made in equal amounts during the period of 5 years. How high should the annuity payments be if the interest rate is 25%?

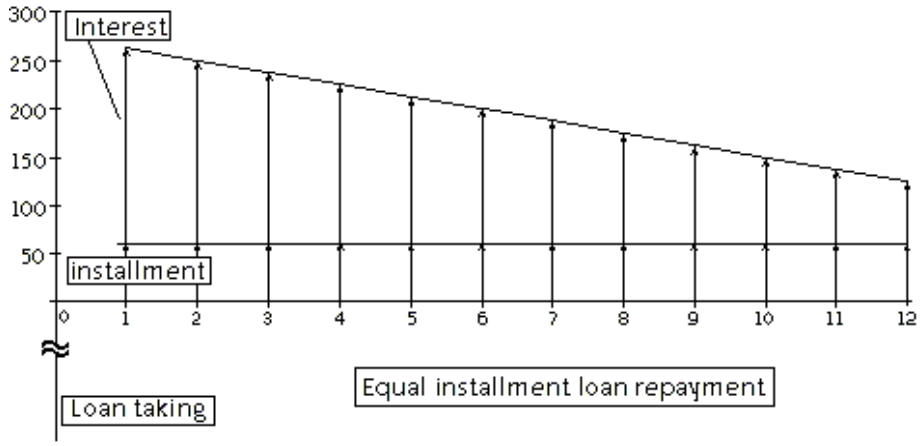
Task 2. An investor is looking for a security that will allow him to gain PLN 50 per year, while the desired rate of return would be 10%. How much should/could he pay for that security?

Task 3. PLN 15 000 was deposited in a bank. After four years, an additional deposit of PLN 500 was made at the end of each quarter. How big will the capital be after 7 years? The bank uses quarterly capitalisation: *r = 12%*.

Task 4. From an account, in which there is an accumulated capital of PLN 100 000, PLN 800 was withdrawn at the beginning of each month. How much capital will be there after 5 years, if *r = 24%*, and the capitalisation is monthly.

6.2. Credit repayment

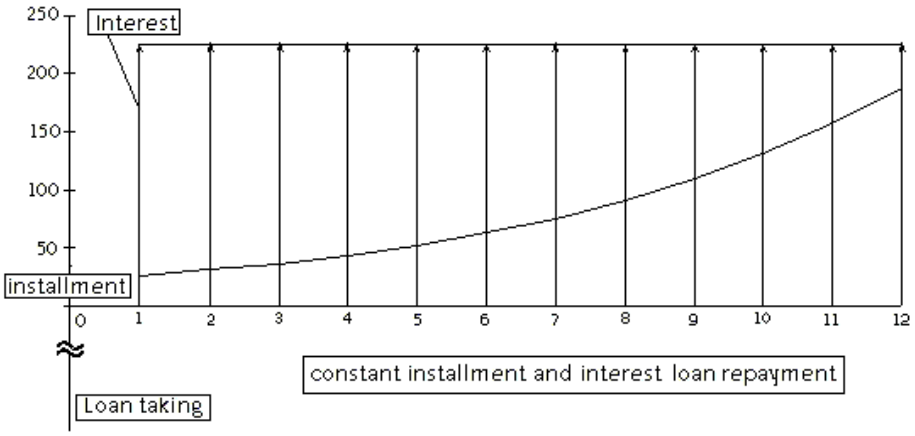
Repayment of capital with equal installments



$$R_i = \frac{K}{n} \qquad I_i = (K - (i-1) * R) * r$$

R_i – amount of i-rate
I_i – amount of interest in the i-th time

Repayment of capital with equal payments – a constant amount of installments and interest



This is a normal annuity, $PVA = PMT * MWOR_n^r$ from where

$$PMT = \frac{PVA}{MWOR_n^r} = \frac{K}{MWOR_n^r}$$

where $MPT = R_i + I_i$.

Task 1. The TOR company has taken a bank loan in the amount of PLN 100 000 for four years. The loan interest rate is 25% per year. What will be the annual installments payable at the end of each year if the company wishes to pay in equal installments of interest and capital? What will be the annual installments payable at the end of each year, if the company wishes to pay equal installments of capital plus interest? Present a loan amortisation table for both cases. Compare the cost of the loan.

Task 2. A loan for PLN 5 000 has to be repaid in 5 equal annual installments with the nominal rate amounting to 12% and an annual interest capitalisation. The loan has a repayment grace period of 4 years. Create a load amortisation table of the loan.

Task 3. A consumer takes a bank loan, which he should repay over the period of 8 years. The loan is for PLN 24 000, with an interest rate of 20% per year. The loan has a repayment grace period of 4 years. Create a repayment plan of this loan in the case of: equal installments and equal payments. Check the cost of the loan. Make a schematic drawing of the relationship between the size of installment and interest for both cases.

Task 4. Calculate the effective annual interest rate for a loan for PLN 10 000 to be repaid over the period of five years in equal monthly installments of PLN 300 at the end of each month if the commission is 5% of the loan value and is paid on the day when the loan is taken.

6.3. Valuation of shares and bonds

Share value:

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+k_s)^t} + \frac{P_n}{(1+k_s)^n}$$

$\sum_{t=1}^{n} \frac{D_t}{(1+k_s)^t}$ – discounted sum of dividends during the share holding,

$\frac{P_n}{(1+k_s)^n}$ – discounted share price that can be obtained from its sale after the holding period.

Share valuation models – constant dividend growth model

Drop in net profit	$k_s > g$; $g < 0$	$P_0 = D_0 \frac{1+g}{k_s - g}$
Steady net profit	$k_s > g$; $g = 0$	$P_0 = \frac{D}{k_s}$
Steady growth in net profit (normal growth of the company)	$k_s > g$; $g > 0$	$P_0 = D_0 \frac{1+g}{k_s - g}$
Unstable growth of the company (supernormal growth of the company in the first n periods)	$k_s < g_1$; $g_1 > 0$ $k_s > g_2$; $g_2 > 0$	$P_0 = \sum_{t=1}^{n} \frac{D_0(1+g_1)^t}{(1+k_s)^t} + \frac{P_{t=n}}{(1+k_s)^n}$ $P_{t=n} = D_n \frac{1+g_2}{k_s - g_2} = D_0(1+g_1)^n \frac{1+g_2}{k_s - g_2}$ $P_0 = \sum_{t=1}^{n} \frac{D_0(1+g_1)^t}{(1+k_s)^t} + \frac{D_0(1+g_1)^n(1+g_2)}{k_s - g_2} \frac{1}{(1+k_s)^n}$

Bond value:

$P_0 = \sum_{t=1}^{T} \frac{d_t}{(1+k_{ob})^t} + \frac{B}{(1+k_{ob})^T}$, where $d_1 = d_2 = \cdots = d_T = i_c * B$

$\sum_{t=1}^{T} \frac{d_t}{(1+k_{ob})^t}$ – sum of discounted bond cash flows (coupon) during the time of their possession,

$\frac{B}{(1+k_{ob})^T}$ – the discounted nominal price of the bonds, obtained at the time of buy-out.

There are three variants of selling bonds (depending on the relationship between the bond rate i_c and the desired bond return rate k_{ob}):

1. $i_c = k_{ob}$, then $P_0 = B$ – bond without a discount
2. $i_c > k_{ob}$, then $P_0 > B$ – bond with a bonus
3. $i_c < k_{ob}$, then $P_0 < B$ – bond with a discount

Task 1. Calculate the value of a four-year bond with a nominal value of 1 000 PLN, with an interest rate of 10%, payable after one year and the market rate of return of:

a) 10%,
b) 8%,
c) 12%.

Task 2. Last year the KORA company paid a dividend to its shareholders in the amount of PLN 2 per share. What is the price per share of the company that we will pay if the expected return rate is 15% in the following situations:

a) The company's net profit will decline at a rate of 5% per year (*g = -5%*),
b) The company's net profit will be maintained at a constant level (*g = 0%*),
c) The company's net profit will grow at a rate of 6% per year (*g = 6%*),
d) We assume that there is an inconstant growth of the company: it is projected that for the first five years the company will increase in a supernormal way 20% (g = 20%), and then the increase will be 5% to infinity.

6.4. Cost of capital

Weighted Average Cost of Capital (WACC)

$$WACC = k_d(1-T)\left(\frac{D}{D+E}\right) + k_e\left(\frac{E}{D+E}\right)$$

Where:
WACC – Weighted Average Cost of Capital,
k_d – pre-tax cost of debt,
T – income tax rate,
$k_d(1-T)$ – post-tax cost of debt,
E – equity value,
D – debt value,
D+E – total capital value (equity + debt),
k_e – equity cost,
$\left(\frac{D}{D+E}\right)$ – share of foreign capital (debt) in total capital
$\left(\frac{E}{D+E}\right)$ – share of equity in total capital.

Weighted Average Rate of Debt (WARD)

$$WARD = i_{d_1} * (1-T) * \frac{D_1}{D} + i_{d_2} * (1-T) * \frac{D_2}{D} + \cdots + i_{d_n} * (1-T) * \frac{D_n}{D}$$

Where:
$WARD$ – Weighted Average Rate of Debt,
$i_{d_1}, i_{d_2}, \ldots, i_n$ – nominal and effective interest on the debt,
$D_1, D_2, \ldots, D_n$ – debt type value,
D – total value of the company debt.

Cost of equity – the capital asset pricing model (CAMP – Capital Asset Pricing Model)

$$k_e = r_f + \beta(r_m - r_f)$$

Where:
k_e – cost of equity capital,
r_f – rate of return on risk-free assets,
β – coefficient of systematic risk for the company,
r_m – rate of return on the market portfolio,
$(r_m - r_f)$ – market risk premium.

Task 1. Calculate the weighted average cost of capital for the company, whose target capital structure is described by the debt to equity indicator (*D/E*) that amounts to 0.80. The rate of return on risk-free assets (long-term treasury bond profit) is currently approx. 6%. The market risk premium is estimated to be 5%, and the company's beta coefficient equals 1.85. The loans taken by the company have an average interest rate of 12% per year. The company pays income tax at a rate of 27%.

Task 2. Calculate the weighted average cost of capital, in which case the assets are financed in 40% by equity. The rate of return on risk-free assets (long-term treasury bond profit) currently amounts to approx. 8%, the rate of return on the market portfolio is 14%, and the company's beta coefficient is 1.55. the total value of loans taken by the company is PLN 500 000. With a loan worth PLN 170 000 and an interest rate of 14%, and a credit for PLN 150 000 – with a rate of 15%, the remaining loans have an interest rate of 12%. The company pays income tax at a rate of 19%.

6.5. Risk pricing

In the case of a portfolio that is comprised of two elements, where the share of security *A* is W_A, and the share of security *B* – W_B with the assumption that $W_A+W_B = 1$:

$$\bar{R}_p = W_A\bar{R}_A + W_B\bar{R}_B$$

Risk of portfolio is measured via the standard deviation:

$$\sigma_{AB} = \sigma_p = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_B\rho_{AB}\sigma_A\sigma_B}$$

σ_{AB}, σ_p – standard deviation of the rate of return, portfolio risk,

σ^2_A, σ^2_B – variation of the rate of return from securities A and B,

σ_A, σ_B –standard deviation of the rate of return from investment in securities A and B,

ρ_{AB} – correlation index between the rates of return from securities A and B.

The linear function is called CML,

$$\overline{R}_p = R_f + \left(\frac{\overline{R}_m - R_f}{\sigma_m} \right) \sigma_p$$

R_f – risk-free rate,
$\overline{\mathrm{R}}_{\mathrm{m}}$ – expected rate of market portfolio – market rate of return,
σ_m – standard deviation of market portfolio,
$\overline{\mathrm{R}}_{\mathrm{p}}$ – expected return from portfolio lying on the CML,
σ_p – standard deviation of the portfolio.

The slope coefficient of this line is the market price of risk.

Market price of risk = $\dfrac{\overline{\mathbf{R}}_{\mathbf{m}} - \mathbf{R}_{\mathbf{f}}}{\boldsymbol{\sigma}_{\mathbf{m}}}$

Task 1.
Stocks of the companies A and B are characterised by the following parameters:

Company	Expected rate of return	Standard dev.
A	30%	10%
B	40%	20%

a) Please find the space of all possible two elements portfolio (*A* and *B*); portfolio risk depending on the proportion of invested capital w_A.

Note: the correlation coefficient takes values from the interval *[-1, +1]*.

b) Please find a set of portfolios; dependency – the rate of return of the expected portfolio and portfolio risk.

Task 2. The rate of return on risk-free assets is currently at approx. 8%, the market rate of return is 15%. The market risk measured by stand. deviation = 20%. Draw the CML for the market. What is the price of the risk at the market?

Task 3. The rate of return on risk-free assets is currently at approx. 5%, the market rate of return is 15%. The market risk measured by stand. deviation = 20% and the company's risk = 30%. Calculate the required rate of return for shareholders. What is the cost of the equity capital for this company?

Task 4. Calculate the weighted average cost of the capital of the TOR Company the activity of which is financed in 40% by equity. The rate of return on risk-free assets is currently at approx. 8%, the market rate of return is 15%. The market risk measured by stand. deviation = 20% and the company's risk = 30%. The total capital of the company is PLN 1 000 000. The company pays 10% of interest of debt and income tax at a rate of 19%.

7. Projects

Project 1.
Plan the financial structure of a company

Project objective: development of the financial structure of a selected enterprise taking into account financial ratios

1. Introduce a company
2. Provide financial statements for the last 3 years (simplified balance sheet as well as profit and loss statements)
3. Make an analysis of the financial structure using appropriate ratios
4. Analyse changes in ratios with changes in the financial structure (debt/equity)
5. Choose and justify the best structure

Project 2.
Estimate the number and price of the shares (stocks) to be issued

Project objective: to estimate the number and price of shares issued by a company that needs a certain amount of equity for investment.

1. Identify the relevant market parameters
2. Determine the requirements of the owners
3. Determine the value of a company
4. Determine the number and price of the newly issued shares
5. Examine the influence of market parameters on the price and number of shares

In the case of the fifth point, prepare a sensitive analysis.
Take into consideration the example of the TOR company (in attachment) and reevaluate the price and number of shares undergoing change.

- The cost of issuing shares
- The rate of return required by the shareholders
- *g* = expected growth rate of the company
- Expected *R* rate of return on invested capital

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