Course: **Capital Budgeting**

Field of study: Finance and Accounting

Form of classes and number of hours: lecture 30 h, practical classes 15 h, project 15 h

Number of ECTS credits: 5

Learning outcomes:
– Knowledge of estimating expenditures and future effects of the investment project and assessment of its financial efficiency.
– Knowledge of issues related to the investment process in the organization.
– Knowledge of preparation of pro-forma statements.
– Knowledge and understanding of methods of investments, financial profitability evaluation, classifications, divisions, purpose of usage.
– Knowledge and understanding of the proper selection of the discount rate in the dynamic methods of investment profitability calculation.
– Ability to build a team for the assessment of the investment project.
– Ability to manage the projected investment, final preparation of pro-forma statements, final financial evaluation of the project.
– Ability to identify the risks associated with the investment.
– Student is aware of the consequences of overly optimistic and too pessimistic estimates of the effectiveness of the proposed investments.
– Student is aware of the responsibility of staff teams for the evaluation of the proposed investment, their accountability for the successes and failures of the company and even the loss of jobs by other employees and bankruptcy of the company.
– Student is aware of the various pressures put on the employees of these teams and the psychological and sociological interactions outside and inside the teams.

Evaluation methods of learning outcomes:
written test, oral exam, discussion, verification of a project

Subject matter of the classes:

1. Introduction to Capital Budgeting – definition of investment, types and stages of investment projects.
2. Estimation of investment cash flows.
3. Static methods of investment profitability calculation.
4. Dynamic methods of investment profitability calculation.
5. Sources of investment financing. Capital costs in estimating the efficiency of the investment. Weighted Average Cost of Capital (WACC).
6. Capital budget optimization.
7. Methods of investment profitability calculation incorporating the element of risk.
8. Investment project's lifetime.
9. Analysis of investment liquidation and replacement. Restitution investments.
10. Separability of investment and financial decisions.
11. The impact of the investment on the company's risk characteristics.

References

[1]   Bräutigam J., Esche Ch., Mehler-Bicher A., *Uncertainty as a key value driver of real options*, 2003, http://www.realoptions.org/papers2003.

[2]   Dotkuś W., 'Inwestycje rzeczowe'. W: *Rachunkowość finansowa i podatkowa*, pod red. T. Cebrowskiej, Wyd. Naukowe PWN, Warszawa 2005.

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[7]   Pindyck R. S., *Irreversibility, uncertainty, and investment*, 'Journal of Economic Literature', Vol. XXIX, September 1991, http://www.samsi.info/sites/default/files/Pindyck91.pdf.

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[9]   Walters Ch., Giles T., *Using real options in strategic decision making*, http://www.mba.tuck. dartmouth.edu/paradigm/spring2000/articles/walters, decision_making.html.

[10]   Wilimowska Z., Szczepańska J., *Selected aspects of economy in industrial practice*, Warsaw University of Technology. Institute of Manufacturing Technology, Warsaw 2005.

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1. Definition of investment and classifications

Investment is the sacrifice of certain current consumption for generally uncertain future consumption. The investor will want to be compensated both for the time that his money is tied up in the investment and for the risk that is involved during that time. The investor expects to be satisfied either by receiving income from his investment or through an increase in the capital value of the investment. The investors require returns that cover their earning expectation and risk. **The rate of return** consists of expected real interest rate, expected inflation rate, expected liquidity premium and expected risk premium. The first three components are the price of time, the fourth is the price of risk.

Investments in financial and tangible assets can be distinguished. **Financial assets** are financial claims assets that are usually documented by some form of legal representation (e.g. bond, share of stock). **Tangible assets** represent actual tangible assets that may be seen, felt, held or collected. Companies invest in a variety of tangible assets such as buildings and machinery and intangible assets such as patents. Investment in tangible assets will be considered as an **investment undertaking.**

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                        │ INVESTMENTS  │
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        ┌───────────────────┬───────────────────┐
┌───────────────┐   ┌───────────────┐   ┌───────────────┐
│   FINANCIAL   │   │   TANGIBLE    │   │  INTANGIBLE   │
└───────────────┘   └───────────────┘   └───────────────┘
    ┌─────────────┐     ┌─────────────┐     ┌──────────────────┐
    │   shares    │     │ real estate │     │   research and   │
    └─────────────┘     └─────────────┘     │   development    │
    ┌─────────────┐     ┌─────────────┐     └──────────────────┘
    │    bonds    │     │new fixed assets│   ┌──────────────────┐
    └─────────────┘     └─────────────┘     │  staff training  │
    ┌─────────────┐     ┌─────────────┐     └──────────────────┘
    │other financial│   │reconstruction│    ┌──────────────────┐
    │ instruments │     │    and      │     │broadly understood│
    └─────────────┘     │modernization│     │      rights      │
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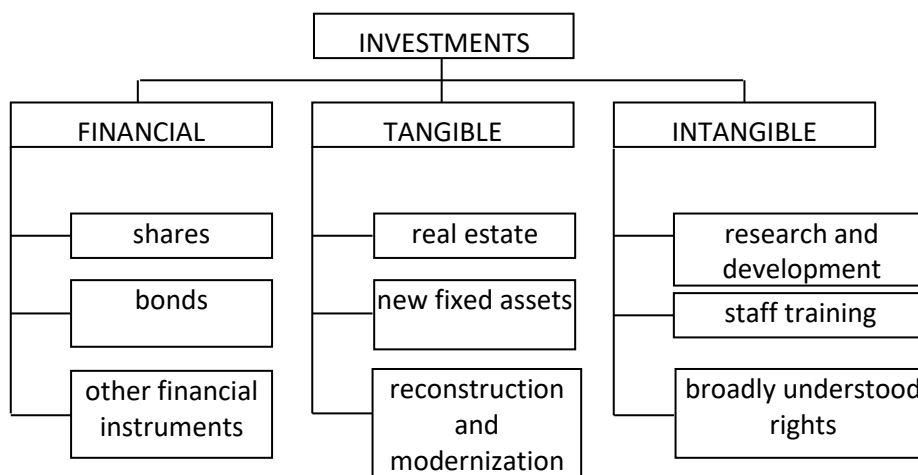
Figure 1.1. Types of investments [2, p. 284]

Investments can also be divided into **reconstruction-, modernization- and new investments** [4].

The purpose of **restitution investments** is not the development of the enterprise or the expansion of the scope of activities conducted so far. They are aimed at recreating decapitalized fixed assets, which in turn allows for the continuation of existence and the maintenance of the company's current position on the market. Failure to make this type of investment may result in a loss of production capacity, because the used fixed assets will prevent production or provision of services. From this point of view, these investments are necessary. They do not affect the development of applied technologies or increase the quality of manufactured products or services. Making only replacement investments leads to the marginalization of the company's position on the market.

The aim of **modernization investments** is not only to rebuild damaged and worn-out fixed assets, but above all to modernize them, i.e. to increase technical and technological quality. Only making this type of investment can provide the company with the conditions to properly compete on the market and defend its position held so far.

The purpose of **new investments** is to increase the production potential, i.e. the efficiency of the machinery and equipment, development of the commercial network, diversification of distribution channels, as well as the implementation of new products for production, satisfying the existing needs of potential buyers better or generating new, previously unknown needs. These investments are aimed at diversifying the conducted activity, and thus reducing the risk of ruining it due to the risk of failure of subsequent projects or the loss of some sales markets. They mainly consist in creating new, previously non-existent components, mainly in the area of fixed assets. This includes the construction of a production hall and investments in new technologies that will increase the quality of products and services, as well as work efficiency. Investments of this type contribute to limiting the costs incurred by the enterprise and have the greatest impact on its development and the possibility of expansion in the market. In the long run, this type of investment should be the most profitable and guarantee the greatest benefits. These are investments that fall within a range of high risk and uncertainty about their success.

Another division is the division of investments into:
- **simple**, i.e. those that do not bring income during the investment; the only income comes at the end of the investment period;
- **coupons**, i.e. those that generate income during the investment, and the income can be consumed or invested.

Another types of division of investments according to the degree of dependence between them can be found in the literature [5]:
- **mutually exclusive**, i.e. those in which the implementation of one investment excludes the implementation of the other,
- **complementary**, i.e. those of which implementation increases the expected profitability of other investments,
- **substitutive**, the acceptance of which reduces the expected profitability of other investments,
- **statistically dependent**, when the increase (decrease) in the benefits of one investment is accompanied at the same time by the increase (decrease) of benefits in the other project, caused by the same phenomenon external to the enterprise.

In addition to the presented classifications, there are also many others, e.g. according to the value of inputs (main, typical, small), according to the degree of necessity (urgent, required, desired) or the degree of risk.

2. Measuring profitability of an investment undertaking

Because the investment outlays are incurred "today" and profits gained thanks to the investment will appear in future – every investment undertaking is risky. Assessment of profitability of an investment undertaking needs very thorough calculations. The problem is that it is not possible to foresee future conditions and investment decision must be taken "today". That is why investors use many evaluation methods to reduce the possibility of investment undertaking failure to minimum.

The most popular methods of valuating investment profitability are based on discounted cash flows. The first basic principle of finance is that the monetary unit today is worth more than the monetary unit tomorrow, because the monetary unit can be invested today to start earning interests immediately. Any investment rule which does not recognize the time value of money cannot be sensible.

The most popular methods are:
- Net Present Value (NPV),
- Internal Rate of Return (IRR),

- Modified Internal Rate of Return (MIRR),
- Payback (PB),
- Discounted Payback (DPB),
- Net Present Value Ratio (NPVR),
- Profitability Index (PI).

2.1. Net Present Value (NPV)

The basic idea behind net present valuation (NPV) is to determine cash flows over the lifetime of a project and discount them by the appropriate opportunity cost of capital. This discount rate should reflect both the time value of money and the risk involved in a given project. If the value is positive (larger than zero), the company should proceed with the project, otherwise it should choose not to invest.

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t},$$

where:

CF_t – cash flow generated by the project during t-period,

k – discount rate (hurdle rate, rate of return, opportunity cost of capital).

The rate of return is the reward that investors demand for accepting delayed payments. It is also called opportunity cost of capital because it is the return forgone by investing in the project rather than investing in securities.

NPV informs how much company market value will grow if the project is implemented. The NPV theory is based on the following assumptions:

- investment outlay is irreversible,
- forecasted structure of cash flows during investment project is known,
- the risk level and corresponding required rate of return are known,
- only exogenous (market) risk is taken into the consideration,
- the only alternative is to invest in the market.

Example 2.1.

Suppose a company is going to invest 1 000 000 EUR in a project. Rate of return is 14%. Lifetime of a project is assessed at 4 years. Forecasted cash flows are 150 000 EUR, 700 000 EUR, 1 030 000 EUR and 840 000 EUR in each year accordingly. Is it worthy to go ahead with implementation of this project?

The answer is 'yes' because NPV>0 i.e. discounted cash inflows exceed investment outlay, Table 2.1.

Managers consider NPV analysis as a shortsighted and incomplete method. An NPV calculation only uses information that is known at the time of the appraisal. NPV assumes the decision to invest in a project is all or nothing. The NPV analysis obliges managers to compute present values of their investment as if they have engaged all the costs. NPV undervalues investment project, because it does not incorporate management flexibility. NPV calculations ignore an investor's ability to modify his behavior in response to incoming information. NPV is frequently applied without careful consideration of the assumptions on which it is based.

Table 2.1. Discounted cash flows calculation

Year	Cash flows [in EUR]	Discounted Cash Flows
0	- 1 000 000	$\dfrac{-1\,000\,000}{(1+0.14)^0} = -1\,000\,000$
1	150 000	$\dfrac{150\,000}{(1+0.14)^1} = 131\,578.95$
2	700 000	$\dfrac{700\,000}{(1+0.14)^2} = 538\,627.27$
3	1 030 000	$\dfrac{1\,030\,000}{(1+0.14)^3} = 695\,220.66$
4	840 000	$\dfrac{840\,000}{(1+0.14)^4} = 497\,347.43$
Total NPV		862 774.31

In the situation when investment conditions are constantly changing it is possible that calculation based on NPV formula can turn out to be insufficient and inadequate to new circumstances. That is why, when it is possible, it is worthy to supplement investment analysis with a **real option valuation**.

And then:
> despite NPV <0,
> but
> NPV + real option value>0
> it would be worthy to go on with a project.

An elevated uncertainty according to investment conditions demands new ways of strategic thinking and new tools for financial analysis. Real options began to attract considerable attention from the industry as a potentially important tool for valuation and strategy.

Real option is the right – but not the obligation – to gain the gross present value of expected cash flows by making an irreversible investment on or before the date the opportunity fails to be available [9].

Real options are not traded on organized stock market exchanges the way that traditional financial options are. They are rather analogous to a "value premise" [1].

There are three main categories of real options:
  − **a timing option** – delay investment until more information or skill is acquired,
  − **an exit option** – shut down a project when there is no further potential in a business opportunity,
  − **a growth option** – follow-up investments if the original project is a success.

To use real options they have to be identified at first. A rich set of criteria is used to recognize the companies' real options. Options can be identified using SWOT analysis that identifies up- and downside potentials. Sometimes uncovering real options can be difficult. Whereas some real options are likely to be incorporated in the project description or the strategy (e.g. patents, ownership of land, natural resources, managerial resources, technological knowledge, reputation, market position, possible scale [7]), others will remain unknown. On the other hand for some uncertainties, no options may exist e.g. armed conflicts. The second step is a real option valuation. A real option

theory has its origins in capital market derivative pricing. A real options approach applies financial option theory. The best known form is the Black-Scholes model. The last step in a real option implementation is the control of a real option valuation, because real options are only valuable if they are exercised accordingly.

2.2. Internal Rate of Return (IRR)

The internal rate of return is defined as the rate of discount which makes NPV = 0.

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1 + IRR)^t} = 0$$

Calculation of IRR usually involves trial and error approach. The easiest way to calculate IRR is to plot few combinations of NPV and discount rate on a graph, connect the points with a smooth line and read off the discount rate at which NPV=0.

Example 2.2.

Suppose we have to find IRR of a project which is supposed to generate the following cash flows in the future: 245 000 EUR, 312 000 EUR, 740 000 EUR and 320 000 EUR. Investment outlay is 1 000 000 EUR.

Table 2.2. NPV values for different discount rates

discount rate	NPV value
0.10000000000000	255 115.77
0.12000000000000	197 557.66
0.14000000000000	143 930.76
0.16000000000000	93 893.55
0.18000000000000	47 139.95
0.19000000000000	24 907.48
0.20160848528247	0
0.20161000000000	-3.19
0.21160848528247	-20 714.18
0.22000000000000	-37 588.45
0.24000000000000	-76 033.60

$IRR = 0.20160848528247 \approx 20\%$

We can also calculate IRR using the following formula:

$$IRR = k_1 + \frac{NPV_p(k_2 - k_1)}{|NPV_n| + NPV_p},$$

where:
k_1 – lower discount rate,
k_2 – higher discount rate,
NPV_p – positive NPV achieved for lower discount rate k_1,
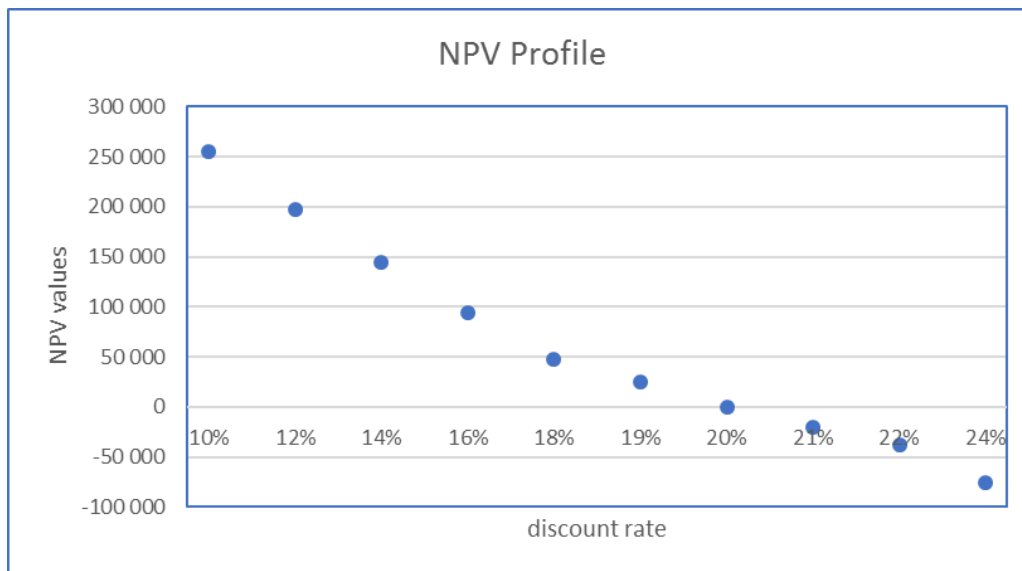NPV_n – negative NPV achieved for higher discount rate k_2.

Figure 2.1. Assessing IRR value

The difference between k_1 and k_2 should not be larger than 1%. Dependence of NPV on k is non-linear and therefore the larger difference between k_1 and k_2 the larger estimation error can be.

The IRR rule is to accept an investment project if the opportunity cost of capital is less than the internal rate of return. It will give the correct answer if properly used. In reality it is not so simple to calculate it. Investor must consider the following situations:

- If a project offers positive cash flows followed by negative flows, NPV rises as the discount rate is increased. The project with IRR lower than the opportunity cost of capital should be accepted then.
- If there is more than one change in the sign of the cash flows, the project may have several IRRs or no IRR at all.
- The IRR rule requires a comparison of the project's IRR with the opportunity cost of capital. Sometimes the opportunity cost of capital may be different for cash flows in different years of the project implementation.

2.3. Modified Internal Rate of Return (MIRR)

The modified internal rate of return for a series of periodic cash flows is the interest rate that considers both the cost of the investment and the interest received on reinvestment of cash. MIRR better reflects the profitability of a project. IRR assumes the cash flows from the project are reinvested at the IRR, whereas the modified IRR assumes that all cash flows are reinvested at the firm's cost of capital. When MIRR > cost of capital – the project should be accepted. The greater the MIRR the better. MIRR can be calculated using Excel formula or the following equation:

$$MIRR = \sqrt[n]{\frac{\sum_{t=0}^{n} CIF_t (1+k)^{(n-t)}}{\sum_{t=0}^{n} \frac{COF_t}{(1+k)^t}}} - 1 = \sqrt[n]{\frac{TV}{I}} - 1,$$

where:
CIF – cash inflow,
COF – cash outflow,
TV – terminal value,
I – investment value,
k – cost of capital.

Example 2.3.

Suppose we have the following project's cash flows like in the Table 2.3.

Table. 2.3. Forecasted project's cash flows

Year	0	1	2	3	4
CF	-800 000	250 000	300 000	350 000	410 000

Let's calculate TV at first assuming that k = 8%.

Table 2.4. Terminal vale calculation

Year	Cash flows	TV
1	250 000	$250000 \cdot (1.08)^3 = 314\,928$
2	300 000	$300000 \cdot (1.08)^2 = 349\,920$
3	350 000	$350000 \cdot (1.08)^1 = 378\,000$
4	410 000	$410000 \cdot (1.08)^0 = 410\,000$
	TOTAL	1 452 848

$$MIRR = \sqrt[4]{\frac{1\,452\,848}{800\,000}} - 1 = 16.09\% \;;\; \text{MIRR} > \text{k} \rightarrow \text{the project can be accepted.}$$

2.4. Payback (PB)

Companies often require that the initial outlay on any project should be recoverable within some specified cutoff period. The payback period of a project is found by counting the number of years it takes before cumulative forecasted cash flows equal the initial investment.

We can calculate it using the following formula:

$$PB = \left(r_{(n-1)}\right) + \frac{K_t}{CF_t},$$

where:

$r_{(n-1)}$ – number of years prior to full recovery,

K_t – uncovered cost at beginning of full-recovery year,

CF_t – cash flow during full-recovery year.

Example 2.4.

What is the payback period in years for projects A and B? Which project should be chosen if they are mutually exclusive projects?

Table 2.5. Cash flows and cumulative cash flows of project A [in EUR]

Year	0	1	2	3	4	5
CF$_A$	-40 000	28 000	4 000	4 000	32 000	32 000
Cumulative CF$_A$	-40 000	-12 000	-8 000	-4 000	28 000	60 000

Table 2.6. Cash flows and cumulative cash flows of project B [in EUR]

Year	0	1	2	3	4	5
CF$_B$	-40 000	8 000	16 000	16 000	4 000	4 000
Cumulative CF$_B$	-40 000	-32 000	-16 000	0	4 000	8 000

$$PB_A = 3 + \frac{4\,000}{32\,000} = 3.125 \; years$$

$$PB_B = 2 + \frac{16\,000}{16\,000} = 3 \; years$$

Project B is better in terms of the payback period. Considering the cash distribution in time the investor would prefer project A, because after the first year cash flows cover 70% of investment outlay and what's more after the third year cash flows from project A are greater than from project B.

2.5. Discounted Payback (DPB)

Payback period does not recognize the time value of money. For this reason its practical implementation is rather small. That is why more useful is discounted payback. It recognizes that a monetary unit (e.g. 1 EUR) at the beginning of the payback period is worth more than a monetary unit at the end of the payback period. Because of time value of money implementation discounted payback is longer than undiscounted one.

Example 2.5.
Let's use cash flows from the previous example. Discount rate of return is 12%.

Table 2.7. Cash flows, discounted cash flows and cumulative discounted cash flows of project A [in EUR]

Year	0	1	2	3	4	5
CF_A	-40 000	28 000	4 000	4 000	32 000	32 000
Discounted CF_A	-40 000	25 000	3 188.78	2 847.12	20 336.58	18 157.66
Cumulative discounted CF_A	-40 000	-15 000	-11 811.22	-8 964.10	11 372.48	29 530.13

Table 2.8. Cash flows, discounted cash flows and cumulative discounted cash flows of project B [in EUR]

Year	0	1	2	3	4	5
CF_B	-40 000	8 000	16 000	16 000	4 000	4 000
Discounted CF_B	-40 000	7 142.86	12 755.10	11 388.48	2 542.07	2 269.71
Cumulative discounted CF_B	-40 000	-32 857.14	-20 102.04	-8 713.56	-6 171.48	-3 901.78

$$DPB_A = 3 + \frac{8\,964.10}{20\,336.58} = 3.44 \; years$$

Cash flows of project B are not sufficient to cover investment outlay during the five years of project's lifetime.

2.6. Net Present Value Ratio (NPVR)

When comparing two or more mutually exclusive projects, we often find that investment outlays related to the projects under assessment differ from each other, both in terms of the value of expenditure and the different period incurring these outlays. Thus, judging only on the basis of the NPV criterion may not be sufficient as the NPV only measures the net cash flows. However, it does

not inform exactly about the differences in the level of profitability of competitive projects. Then the Net Present Value Ratio (NPVR) can be used as:

$$NPVR = \frac{NPV}{PVI},$$

where:
NPV – Net Present Value of investment project,
PVI – Present Value of the Investment.

The choice of an investment project is determined by the highest value of the NPVR ratio. This means that the selected project will be the most effective. This indicator is also useful for evaluating and comparing various financing options for a selected project.

Example 2.6.
The company is considering two mutually exclusive projects X and Y of a similar risk level. The cost of capital is 12%. Company cares about the most effective investment of capital. What decision should be made?

Table 2.9. Cash flows of project X and Y [in thous. EUR]

year	0	1	2	3	4	5
CF_x	-100	40	23	35	41	87
CF_y	-56	12	15	20	39	62

$$NPV_x = \frac{-100}{(1+0.12)^0} + \frac{40}{(1+0.12)^1} + \frac{23}{(1+0.12)^2} + \frac{35}{(1+0.12)^3} + \frac{41}{(1+0.12)^4}$$

$$+ \frac{87}{(1+0.12)^5} = 54.38 \ thous. EUR$$

$$NPV_y = \frac{-56}{(1+0.12)^0} + \frac{12}{(1+0.12)^1} + \frac{15}{(1+0.12)^2} + \frac{20}{(1+0.12)^3} + \frac{39}{(1+0.12)^4}$$

$$+ \frac{62}{(1+0.12)^5} = 40.87 \ thous. EUR$$

$$NPVR_x = \frac{54.38 \ thous. EUR}{100 \ thous. EUR} = 0.54$$

$$NPVR_y = \frac{40.87 \ thous. EUR}{56 \ thous. EUR} = 0.73$$

Comparing NPV values, project X is better, but when we look at NPVR, project Y is more effective and this project should be accepted.

2.7. Profitability Index (PI)

It can be used for evaluation of a single project and for choosing the best one out of the proposed projects. It is calculated on the basis of the following formula:

$$PI = \frac{\sum_{t=1}^{n}\frac{CIF}{(1+k)^t}}{\sum_{t=1}^{n}\frac{COF}{(1+k)^t}} = NPVR + 1,$$

where:
CIF – cash flows generated by the project,
COF – investment outlays.

Example 2.7.
Let's calculate PI of the projects X and Y from the previous example.

$$PI_x = \frac{\frac{40}{(1+0.12)^1} + \frac{23}{(1+0.12)^2} + \frac{35}{(1+0.12)^3} + \frac{41}{(1+0.12)^4} + \frac{87}{(1+0.12)^5}}{\frac{100}{(1+0.12)^0}} = 1.54$$

$$PI_y = \frac{\frac{12}{(1+0.12)^1} + \frac{15}{(1+0.12)^2} + \frac{20}{(1+0.12)^3} + \frac{39}{(1+0.12)^4} + \frac{62}{(1+0.12)^5}}{\frac{56}{(1+0.12)^0}} = 1.73$$

If projects X and Y are independent, both are acceptable, because $PI_x > 0$ and $PI_y > 0$.
If these projects are mutually exclusive, project Y is better than project X, because $PI_y > PI_x$.

3. Measuring profitability of an investment undertaking including risk factor

In risky situations the investor using NPV analysis has to set the discount rate with regard to the higher risk level. The company cost of capital is the correct discount rate for projects that have the same risk as the company's existing business. In principle, each project should be evaluated at its own opportunity cost of capital. When investor estimates the cost of capital for a particular project, project risk must be included in its calculation.

There are methods that regard risk:
 – directly,
 – and indirectly.

The methods regarding risk directly are:
 – Certainty Equivalent Adjustment (CEA),
 – Risk – Adjusted Discount Rate Approach (RADRA).

3.1. Certainty Equivalent (CE)

Certainty equivalent (CE) is the amount that can be gained with no risk and what is the most important, it has the same utility as the higher but uncertain amount. Uncertain cash flows generated by the project are replaced with lower but risk-free cash flows.

$$NPV = \sum_{t=0}^{n} \frac{a_t CF_t}{(1+k_{RF})^t},$$

where:
a_t – certainty equivalent factor, $a_t \in [0,1]$,
k_{RF} – risk-free discount rate.

Decreasing value of certainty equivalent factor is being taken into account for the following years of the project realization. CE reflects subjective attitude to the uncertain future conditions. In this way decision-makers can express their experience, intuition and risk attitude since CE is higher for the optimists than for the pessimists.

Example 3.1.
Suppose we have the following certainty equivalents and project's cash flows.

Table 3.1. Project's cash flows calculation including certainty equivalent factor [in EUR]

Year	0	1	2	3	4	5
CF_t	-400 000	60 000	55 000	64 000	71 000	85 000
a_t	1	0.95	0.9	0.85	0.8	0.75
$a_t \cdot CF_t$	-400 000	57 000	49 500	54 400	56 800	63 750

When we use k_{RF}=5% to discount cash flows we will get NPV = -157 144.27 EUR. The project cannot be accepted.

3.2. Risk Adjusted Discount Rate Approach (RADRA)

RADRA is based on discount rate correction. The discount rate should reflect the degree of risk so when the project is considered to be riskier, the discount rate (WACC) should be appropriately adjusted. It can be calculated from the capital asset pricing model. CAPM states that the required return on any asset depends on its risk.

$$RADR = k = k_{RF} + \beta(k_M - k_{RF}),$$

where:
$RADR$ – risk adjusted discount rate,
k – required rate of return,
k_{RF} – risk-free interest rate,
k_M – rate of return on the market,
β – a stock's sensitivity to changes in the value of the market portfolio (Beta measures the marginal contribution of a stock to the risk of the market portfolio),
$(k_M - k_{RF})$ – market risk premium.

The most common way to estimate the beta of a stock is to figure out how the stock price responded to market changes in the past. A more reliable figure can be obtained when investor calculates an industry beta for a group of similar companies. It is not enough to estimate beta and plug it into CAPM, because stock beta may reflect both business and financial risk. When a company borrows money, it increases beta and at the same time it increases the expected return of company's stock. Company cost of capital is the expected return on a portfolio of all the company's securities, not just the common stock. It can be calculated by estimating the expected return on each of the securities and then taking a weighted average of these separate returns.

Example 3.2.

Expected cash flows of the project are given in Table 3.2. The company's cost of capital is 8%. The risk-free interest rate is 5%, the market risk premium is 7%, and project X's beta is 1.2. Let's calculate project's NPV using the company cost of capital and RADR as the discount rate.

$$RADR = k = 5\% + 7\% \cdot 1.2 = 13.4\%$$

Table 3.2. NPV calculation [in thous. EUR]

Year	Cash flow	PV of cash flow at 8%	PV of cash flow at 13.4%
0	-150	-150.00	-150.00
1	92	85.19	81.42
2	55	47.15	43.07
3	35	27.78	24.26
Total PV		**10.12**	**-1.25**

If investor used the company cost of capital in NPV calculation, the project would be accepted. This procedure can cause that unprofitable, risky projects will be accepted. Only when RADR is used as the discount rate the right value of the project NPV is calculated.

Methods that regard risk indirectly:
- scenario analysis,
- decision tree,
- sensitivity analysis,
- Monte Carlo simulation.

3.3. Scenario Analysis

Scenario analysis constructs a number of scenarios i.e. vivid stories of a possible future states of the world, that arise from different resolutions of the key uncertainties present in the environment. A scenario is a description of the current situation as well as of a series of events that could lead from a current to a future scenario. Scenario analysis method involved forward projection which allows us to progress from the current situation to a series of alternative, future scenarios. Most often three scenarios are considered: optimistic, best and pessimistic. As a result we get three values of NPV. If we know probability distribution of these scenarios, the expected value of NPV can be calculated using the following formula:

$$E(NPV) = NPV_O \cdot p_O + NPV_B \cdot p_B + NPV_P \cdot p_p,$$

where:
NPV_O – NPV of optimistic scenario,
p_O – probability of optimistic scenario appearance,
NPV_B – NPV of best scenario,
p_B – probability of best scenario appearance,
NPV_P – NPV of pessimistic scenario,
p_P – probability of pessimistic scenario appearance.

It is also possible to calculate standard deviation that stands for the level of project risk.

$$s(NPV) = \left[p_B \cdot \left(NPV_B - E(NPV) \right)^2 + p_O \cdot \left(NPV_O - E(NPV) \right)^2 + p_P \cdot \left(NPV_P - E(NPV) \right)^2 \right]^{1/2}$$

If we had two or more mutually exclusive projects we could also calculate volatility index (VOLI) which measures how many risk units fall per unit of the expected value, i.e.:

$$VOLI = \frac{s(NPV)}{E(NPV)}.$$

The lower the value of volatility index, the better.

Example 3.3.
Suppose we have the following scenarios of the project.

Table 3.3. Project's scenarios [cash flows in thous. EUR]

Scenarios	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Probabilities
Optimistic scenario (O)	-100	34	38	43	47	54	0.3
Best scenario (B)	-100	25	29	34	38	45	0.5
Pessimistic scenario (P)	-100	16	20	25	29	36	0.2
							$\sum 1$

When k = 9% we get:
$$NPV_O = 64.77 \; thous. EUR$$
$$NPV_B = 29.77 \; thous. EUR$$
$$NPV_P = -5.24 \; thous. EUR$$
$$E(NPV) = 33.27 \; thous. EUR$$
$$s(NPV) = 24.50 \; thous. EUR$$
$$VOLI = 0.74$$

0.74 unit of risk fall per one unit of the expected value.

Even when $NPV_P < 0$ the project is acceptable, because $E(NPV) > 0$. The pessimistic scenario might not come true.

3.4. Decision Tree Approach

Decision tree approach is used for complicated staged projects when the decision taken earlier has its implication for future situation. The sequence of events is very important here. This is the extended scenario analysis. Decision-makers have to fix investment outlay, expected cash flows and probability of every possible situation. The expected value of the project and standard deviation as a risk measure can be calculated here.
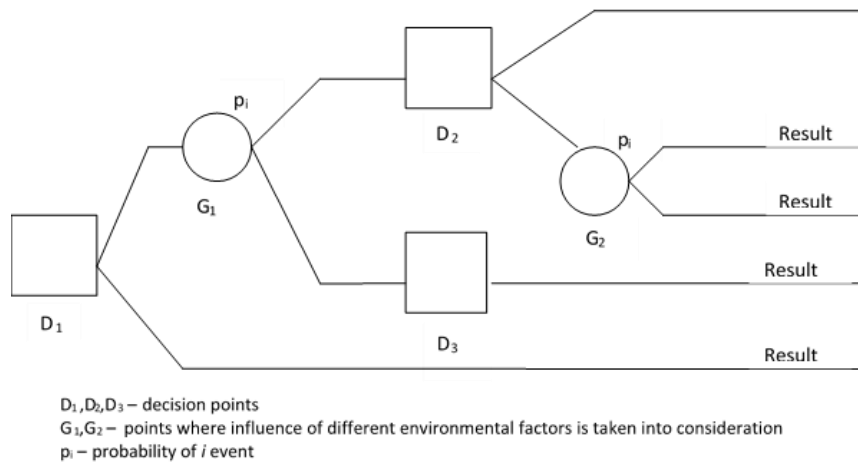
D₁,D₂,D₃ – decision points
G₁,G₂ – points where influence of different environmental factors is taken into consideration
pᵢ – probability of *i* event

Figure 3.1. Example of a decision tree [8, p. 164]

3.5. Sensitivity Analysis

Sensitivity analysis measures the impact on project outcomes of changing key input value about which there is uncertainty. An analysis is performed to see how the outcome changes as chosen value is considered in turn, with other things kept the same. In engineering economics, sensitivity analysis measures the economic impact resulting from alternative values of uncertain variables that affect the economics of the project. When computing measures of project worth, for example, sensitivity analysis shows just how sensitive the economic payoff is to uncertain values of a critical input, such as the discount rate or project maintenance costs. Sensitivity analysis reveals how profitable or unprofitable the project might be if input values to the analysis turn out to be different from what is assumed in a single-answer approach to measuring project worth. Of course, the less sensitive the project is the better.

Example 3.3.
Suppose we have the following cash flows like in Table 3.4.

Table 3.4. Project's cash flows [cash flows in thous. EUR]

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
-100	25	32	45	55	60

When k = 12% →NPV = 48.86 thous. EUR.

Supposing that investment outlay increased by 10%, NPV is equal to 38.86 thous. EUR.

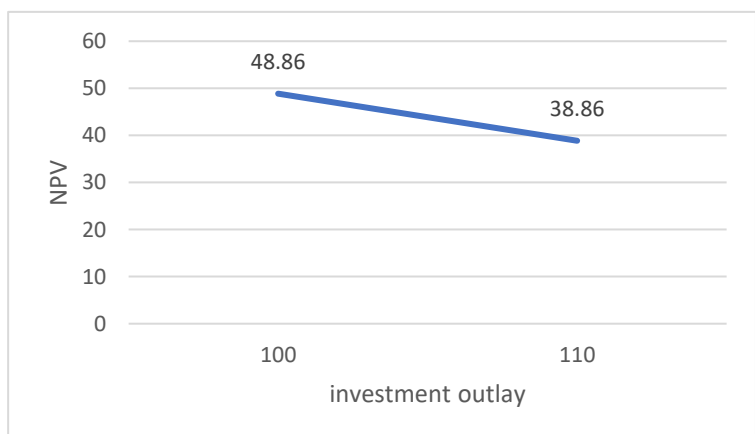10% change of investment outlay caused 20.47% change of NPV.

Figure 3.2. Project's sensitivity to change in investment outlay

3.6. Monte Carlo Simulation

Monte Carlo method is for studying the statistical confidence limits in distribution analyses. The idea of the Monte-Carlo analysis is the generation of a large number (e.g. $100 - 1000$) of synthetic data sets that are similar to the experimental data set, but each one with different random normally distributed noise. Each of these new data sets is analyzed, and the distributions are stored. The resulting set of distributions can then be studied, point by point, and the mean and probability contours can be calculated. In essence, the Monte-Carlo statistics allows to build up a probability distribution function, and allows to study how much the noise of the data is translated into the distribution. As a consequence, the Monte-Carlo procedure can be useful for investigating what features of the distribution are a result of noise in the data, and what features are essential for describing the data. The underlying assumption made in the current implementation is that the best-fit distribution represents the 'true' data well enough, so that the effects of noise in the simulated data and in the real data are the same. A second assumption made is that the error distribution of each data point is Gaussian one. So, thanks to computer simulations we can get the probability distribution of project NPV. The expected value and standard deviation of the project are the final results of this simulation.

4. Incremental Cash flows

Effectiveness of investment project is different when we consider investment as an individual undertaking and is different when we consider an investment as a part of the whole company's activity. Therefore, to assess appropriate effectiveness of the considered investment, we have to assess adequate cash flows as of the date before investment was implemented and adequate cash flows as of the date when investment has already been introduced. Then we have to assess incremental cash flows.

Incremental cash flow is defined as the difference between appropriate amounts after and before the implementation of the investment for each period of time $t = 1,2, \dots, n$.

$$NCF_t = \triangle EBT_t + \triangle D_t = \triangle NWC_t(1 - T) + \triangle D_t + \triangle NWC_t,$$

where:
ΔEBT – change in Earnings Before Taxes (EBT),
ΔD – change in Depreciation,
ΔEAT – change of Earnings After Taxes (EAT),
ΔNWC – change in Net Working Capital (NWC),
T – income tax rate,
$t = 1,2, \dots, n$ – number of investment period.

or

$$NCF_t = (\triangle S_t - \triangle O_t - \triangle D_t)(1 - T) + \triangle D_t + \triangle NWC_t,$$

where:
$\Delta S = S_2 - S_1$ – change in revenue,
$\Delta O = O_2 - O_1$ – change in operating costs,

and indexes mean:

2 – situation after implementing the project,
1 – situation before implementing the project.

Example 4.1.

Company X decided to expand its activity by opening a new store. In order to complete the project an additional room has to be rented and equipped. Equipment cost is 60 000 EUR. Transportation cost and equipment installation cost is 6 000 EUR. Forecasted increase in working capital – 8 000 EUR.

After the launch of the investment the forecasted increase in revenue is: in the first year – 45 000 EUR, in the second year – 50 000 EUR, in the third year – 65 000 EUR, in the fourth year – 70 000 EUR and in the fifth year – 60 000 EUR. Expected operating costs including rent payment in the first year – 25 000 EUR. It is expected that operating costs will be increasing each year by 8% through the 5 years of project's lifetime. Linear depreciation of equipment will be used and equipment value after the 5 years of the project's lifetime will equal zero. Rate of income tax is 32%.

Investment outlay is calculated in the following manner:

NINV = equipment cost + transportation cost + increase in working capital,

NINV = 74 000 EUR.

Calculation of incremental cash flows of the project is presented in Table 4.1.

Table 4.1. Calculation of annual cash flows NCF_i for Company X

in [EUR]	year 1	year 2	year 3	year 4	year 5
Change in revenue	45 000	50 000	65 000	70 000	60 000
- change in operating costs	25 000	27 000	29 160	31 493	34 012
- change in depreciation	13 200	13 200	13 200	13 200	13 200
= change in EBT	6 800	9 800	22 640	25 307	12 788
- tax	2 176	3 136	7 245	8 098	4 092
=change in EAT	4 624	6 664	15 395	17 209	8 696
+ depreciation	13 200	13 200	13 200	13 200	13 200
+ equipment value after the lifetime					0
+ return of working capital					8 000
CF$_i$	17 824	19 864	28 595	30 409	29 896

Therefore NCF_i for the project can be presented as:

[in EUR]	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
NCF_i	-74 000	17 824	19 864	28 595	30 409	29 896

Every investment project can be divided into four stages that have different character and distribution of financial expenditures in time [3]. These stages are:

– pre-investment stage,
– stage of investment project inception,
– stage of operating,
– stage of investment liquidation.

The way of calculating cash flows depends on the particular stage.

Stage of investment projects inception

Example 4.2.

In this case all expenses are borne at the same time.

Company is going to launch production of a new product which will be manufactured in a new plant. It is anticipated that expenses are (in thous. EUR): purchase of land – 500, new production hall building – 200, installation of new technical equipment and machinery – 150. In order to launch a new production line it will be necessary to increase current assets by 30 thous. EUR and current liabilities by 8 thous. EUR. Recruitment and training of new employees will require 34 thous. EUR. Estimated advertising costs – 55 thous. EUR. The rate of income tax – 32%.

Costs of advertising, training of employees are costs of gaining receipts.
Therefore Tax savings = 0.32 · (advertising costs + recruitment and training costs) = 28 thous. EUR

Thus the actual amount of expenses for advertising and training is:
89 thous. EUR – 28 thous. EUR = 61 thous. EUR

$NCF_0 = NINV$ = capital expenditure + net working capital + actual expenses of advertising and training = 850 thous. EUR + 22 thous. EUR + 61 thous. EUR = 933 thous. EUR

The way of calculating net investment (NINV) is presented in Table 4.2.

Table 4.2. Net cash flow in the stage of investment project inception

Type of operation	Kind of expenditure	Value [thous. EUR]
	Land	500
+	Buildings	200
+	Machinery	150
=	Capital expenditures	850
	Current assets	30
-	Current liabilities	8
=	Net working capital	22
	Advertising costs	55
+	Recruitment and training costs	34
=	Total costs of advertising, recruitment and training	89
-	Tax savings on costs of advertising, recruitment and training	28
=	Net costs of advertising, recruitment and training	61
$NCF_0 = NINV$		933

In case when investment outlays are incurred in several succeeding periods we can transfer them to one period of time calculating their present value at t = 0 or calculating their future value at the moment of commencement of operating.

Example 4.3.

Inception stage lasts four years. Table 4.3. contains the examples of investment expenditures.

Figures contained in the table mean that, for example the company will be incurring land purchase expenditures for four years i.e. in the first year 100 thous. EUR and in the following years 50 thous. EUR.

Table 4.3. NCF_t ($NINV_t$) in the following years of project inception

Type of operation	Kind of expenditure	Value [thous. EUR]			
		in the particular time periods			
		0	1	2	3
	Land	100	50	50	50
+	Buildings	60	0	62	60
+	Machinery	0	0	120	300
=	Capital expenditures	160	50	232	410
	Current assets				21
-	Current liabilities				5
=	Net working capital	0	0	0	16
	Advertising cost			30	60
+	Recruitment and training costs			15	21
=	Total costs of advertising, recruitment and training	0	0	45	81
-	Tax savings on costs of advertising, recruitment and training	0	0	14	26
=	Net costs of advertising, recruitment and training	0	0	31	55
=	Net cash flows	160	50	263	481

Variant I

Net cash flows (NCF) from all stages of capital investment are discounted at the moment of the first investment outlay t = 0. The present value of total net cash flows from particular periods in the inception stage is then as follows:

	year 0	year 1	year 2	year 3
NCF_t [in thous. EUR]	160	50	263	481
Present Value Interest Factor for k = 0.12 $$PVIF = \frac{1}{(1+k)^t}$$	1.0000	0.8929	0.7972	0.7118
$PV(NCF_t)$ [in thous. EUR]	160	45	209	342
Total $PV(NCF_t)$ [in thous. EUR]	756			

Variant II

The value of cash flows from particular periods is calculated at the moment of inception of operating stage.

	year 0	year 1	year 2	year 3
NCF_t [in thous. EUR]	160	50	263	481
Compound Value Interest Factor for k = 0.12 $$CVIF = (1+k)^t$$	1.4049	1.2544	1.1200	1.0000
$FV(NCF_t)$ [in thous. EUR]	225	63	295	481
Total $FV(NCF_t)$ [in thous. EUR]				1 063

In the **stage of operating** the cash flows are settled in two steps. First step is Earnings After Taxes (EAT) calculation. In the second step: depreciation is added to EAT, changes in net working capital (NWC) and alternatively deferred taxes are included.

Example 4.4.

Assessing NCF in the fourth year of the investment operating stage.

The company bought machinery and equipment for 350 thous. EUR. They will be used for 10 years. Their value after 10 years will be equal to 0. Linear depreciation will be used for tax purposes. "Life-time" of the equipment is set for 5 years. In the analyzed year, Earnings Before Depreciation and Taxes (EBDT) are equal to 180 thous. EUR. It is assumed that due to the investment – current assets will increase by 70 thous. EUR and current liabilities by 26 thous. EUR. Rate of income tax is 32%.

depreciation installment for tax purposes is:
0.2 · 350 thous. EUR = 70 thous. EUR

depreciation installment for accounting purposes is:
0.1 · 350 thous. EUR = 35 thous. EUR

net working capital = current assets – current liabilities =
= 70 thous. EUR – 26 thous. EUR = 44 thous. EUR.

increase in deferred taxes = (depreciation installment for tax purposes – depreciation installment for accounting purposes)·income tax rate = (70-35) · 0.32 = 11.2 thous. EUR

The following steps of calculation of cash flow in the fourth year of the investment operating stage are shown in Table 4.4.

Table 4.4. Estimated net cash flows in the first year of the investment operating stage (the fourth year of the project)

Type of operation	Specification	Income for tax purposes [thous. EUR]	Income for accounting purposes [thous. EUR]
	Earnings Before Depreciation and Taxes (EBDT)	180	180
-	Depreciation	70	35
=	Earnings Before Taxes (EBT)	110	145
-	Tax	35.2	46.4
=	Earnings After Tax (EAT)	74.8	98.6
+	Depreciation	70	35
-	Increase of Net Working Capital (NWC)	44	44
+	Increase in deferred taxes	0	11.2
=	NCF$_4$	100.8	100.8

In the **stage of investment liquidation** there are two groups of revenue:
- revenue on fixed assets sales,
- revenue resulting from net working capital liquidation (sale of stock, collection of account receivables, payments of account liabilities).

Example 4.5.

Investment liquidation stage – NCF_i in the liquidation stage

The company assumes that after 10 years the building and the land will be sold. The selling price of land will be equal to its purchase price i.e. 250 thous. EUR. In 10 years the building will not be fully depreciated. Its book value will be 50 thous. EUR. However the company will sell it for 97 thous. EUR.

Full depreciation of machinery and equipment is assumed. After 10 years the value of current assets will be equal to 64 thous. EUR. The value of irrecoverable account receivables is assessed at 8 thous. EUR and value of current liabilities 36 thous. EUR. Income tax rate is 32%.

The difference resulting from the sale of building: selling price – book value, will be taxed:
0.32 · (97 thous. EUR – 50 thous. EUR) = 15.04 thous. EUR.

Tax savings from irrecoverable account receivables are equal to 2.56 thous. EUR (0.32 · 8 thous. EUR).

The method of calculation of final NCF_t for the example in consideration is shown in Table 4.5. Having calculated CF_t in every stage of investment project we can now choose the method of evaluating investment profitability.

Table 4.5. Net cash flow in the liquidation stage

Type of operation	Kind of inflow	Value [in thous. EUR]
	Land	250
+	Buildings	97
-	Tax	15.04
=	Capital inflow	331.96
	Current assets	64
-	Tax savings	2.56
-	Current liabilities	36
=	Net working capital	25.44
=	NCF	357.4

The value of cash flows of investments that have already been completed is known. In case of investment projects that are in the stage of planning, the value of cash flows is a forecasted value so it is not a value that will be received for sure. Uncertainty refers to future revenue as well as to costs. For this reason these cash flows are considered as risky cash flows. It is often assumed that cash flows are random variables having appropriate probability distribution.

5. Weighted Average Cost of Capital (WACC)

Weighted average cost of capital (WACC) is a calculation of a company's cost of capital that weights each category of capital proportionately. WACC calculation includes all capital sources, such as common stock, preferred stock, bonds and any other long-term debt. WACC is simply the weighted average expected cost for the company's various types of obligations – debt, preferred stock, and common stock – that are issued by the corporation to finance its operations and investments.

Broadly speaking, the assets of a company are financed by either debt or equity. WACC is the average of the cost of each of these sources of financing weighted by their respective usage in the given situation. By taking a weighted average, we can see how much interest the company has to pay for every monetary unit it borrows. A company's WACC is the overall required return on the company as a whole. It is the appropriate discount rate to use for cash flows similar in risk to the overall company. In an analysis of a potential investment by the company, investment projects that have an expected return that is greater than the company's WACC will generate additional free cash flow and will create positive net present value for stockholders. These corporate investments should result in an increase in stock prices. Investments that earn less than the company's WACC will result in a decrease in stockholder value and should be avoided by the company.

The general expression for calculating the weighted average cost of capital is as follows:

$$WACC = \frac{D}{D+E} \cdot k_d + \frac{E}{D+E} \cdot k_e,$$

where:
D – debt,
E – equity,
k_e – cost of equity,
k_d – cost of debt.

Table 5.1. Computing component costs

COST OF EQUITY:
1. Cost of preferred stock $$k_p = \frac{D_p}{P_p},$$ or $$k_p = \frac{D_p}{P_{pn} - F_{pn}}$$ where: k_p – cost of preferred stock, D_p – preferred dividend per stock, P_p – price of preferred stock, P_{pn} – sale price of preferred stock, F_{pn} – costs of floating a new issue per stock.

2. Cost of internal common equity:	
2.1. Dividend capitalization model approach	The cost of equity for the no-growth case $$k_e = \frac{D}{P},$$ where: k_e – cost of equity, P – stock price, D – dividend.
	The case of perpetual dividends growing at a constant rate $$k_e = \frac{D_0(1+g)}{P} + g,$$ where: g – growth rate.
	The case of dividends growing at a variable rate $$P_0 = \sum_{t=1}^{m} \frac{D_0(1+g_1)^t}{(1+k_e)^t} + \frac{P_m}{(1+k_e)^m} =$$ $$= \frac{\sum_{t=1}^{m} \frac{D_0(1+g_1)^t}{(1+k_e)^m} \cdot D_{m+1}}{k_e - g_2},$$ where: g_1 – above-normal growth rate expected by investors and security analysts over the first m years', g_2 – normal, perpetual, expected growth rate beginning in year *m+1*. The cost of equity k_e can be estimated by solving this equation by trial and error.
2.2. based on Capital Asset Pricing Model – CAPM	$$k_e = k_{RF} + (k_M - k_{RF})\beta,$$ where: k_{RF} – risk-free interest rate, k_M – market rate of return, $(k_M - k_{RF})$ – market risk premium, β – a stock's sensitivity to changes in the value of the market portfolio. Beta measures the marginal contribution of a stock to the risk of the market portfolio.

2.3. based on Arbitrage Pricing Model – APM	$$k_e = k_{RF} + \beta_1(k_1 - k_{RF}) + \beta_2(k_2 - k_{RF}) +$$ $$+ \cdots + \beta_n(k_n - k_{RF}),$$ where: $1, 2, \ldots, n$ – factors that affect the stock's return, $\beta_1, \beta_2, \ldots, \beta_n$ – sensitivity coefficient of i-stock to certain factors $1, 2, \ldots, n,$ $(k_1 - k_{RF}), (k_2 - k_{RF}), \ldots, (k_n - k_{RF})$ – risk premiums associated with certain factors $1, 2, \ldots, n.$

3. Cost of external common equity

$$k_e = \frac{D_0(1+g)}{P-W} + g,$$

where:
W – cost of floating a new issue per stock.

The cost of new capital is higher because:
   − cost of floating a new issue must be covered and
   − the new issue must be underpriced to attract buyers.

From this reason many companies try to avoid this means of raising capital. Its usage depends on company's investment opportunities.

	COST OF DEBT:
1. cost of credit	$$k_d = i_d(1-T),$$ where: k_d – cost of debt, i_d – interest rate, T – tax rate.
2. cost of bonds	$$k_o = \frac{O}{V_0}(1-T),$$ where: k_o – cost of bonds, O – interest calculated on the nominal value of the bond, V_o – market value of the bond.
3. cost of lease	$$k_l = i_l(1-T),$$ where: k_l – cost of lease, i_l – lease interest rate.

Example 5.1. [6]

TOR Company considers future investments projects. The company's schedule of potentially acceptable capital expenditure projects (defined by the management as projects having internal rate of return IRR > 10%) for the next year as follows:

Table 5.1. Project's characteristics

Project's name	Required investment capital [in million dollars]	Internal rate of return IRR [%]
A	4.0	13.8
B	8.0	13.5
C	6.0	12.5
D	5.0	12.0
E	8.0	11.0
F	4.0	10.0

The company's current capital structure (as well as its targeted future capital structure) consists of:
- 40% debt,
- 10% preferred stock,
- 50% common equity.

The company can raise:
- debt $\leq$ $5 million at a pretax cost of 9%,
- debt >$5 million at a pretax cost of 10%,
- preferred stock at an after-tax cost of 10%.

TOR Company expects to generate $10 million of retained earnings over the coming year. Its present dividend rate is D_0 = $2 per share. The company's common stock now is selling at $25 per share ($P_0$ = $25), and new common stock can be sold to net the company $24 per share ($P_0'$ = $24). Forecasted growth rate is 7%. Tax rate is 40%.

I. Cost of capital calculation for each individual component

1) Cost of debt k_d

$k_d = i_d (1 - t) = 9\% \cdot 0.6 = 5.4\%$ for debt $\leq$ $5 mln

$k_d' = i_d' (1 - t) = 10\% \cdot 0.6 = 6\%$ for debt $>$ $5 mln

2) Cost of preferred stock

k_p = 10 % (given)

3) Cost of equity:
- Internal (retained earnings - $10 million)

$k_e = D_0 (1+g) / P_0 + g = \$2(1 + 0.07) / \$25 + 0.07 = 15.6\%$,
- External (new common stock – amounts above $10 million)

$k_e' = D_0 (1+g) / P_0' + g = 15.9\%$.

II. Computation the weighted (marginal) cost of capital for each increment of capital raised

TOR Company raises funds in proportion to its target capital structure from its lowest cost sources first. Here, these sources are: retained earnings (15.6% after-tax cost), preferred stock (10%) and the first $5 million in debt (5.4% after-tax cost).

$$WACC_1 = k(W_1) = 0.40 \cdot 5.4\% + 0.1 \cdot 10\% + 0.5 \cdot 15.6\% = 10.96\%$$

How much total financing (retained earnings, preferred stock, debt) can be obtained before the $5 million in low cost debt is exhausted and TOR company must acquire additional debt funds at the higher cost? Since we know that the target capital structure consists of 40% debt, the total financing, W_1, that this will support, is equal to the amount of low cost debt available divided by the debt fraction in the capital structure:

$$W_1 = \frac{Amount\ of\ low\ cost\ debt\ available}{Debt\ fraction\ of\ capital\ structure} = \frac{\$5\ million}{0.4} = \$12.5\ million$$

This amount consists of: $5 million debt (40% of the total), $1.25 million (10% of the total) and $6.25 million retained earnings (50% of the total).

The amount of $12.5 million represents a break point in the marginal cost of capital schedule. Beyond $12.5 million in total financing, the weighted (marginal) cost of capital will rise, because higher cost debt must now be used. Beyond the first $12.5 million in funds, TOR Company can use higher cost debt (6% after-tax cost), preferred stock (10% after-tax cost) and retained earnings (15.6% after-tax cost) until all retained earnings are exhausted. Out of the total amount of retained earnings available ($10 million), $6.25 million has been used, leaving $3.75 million.

$$WACC_2 = k(W_2) = 0.4 \cdot 6\% + 0.1 \cdot 10\% + 0.5 \cdot 15.6\% = 11.2\%$$

The total financing that can be supported with the retained earnings is as follows:

$$W_2 = \frac{Amount\ of\ retained\ earnings\ available}{Equity\ fraction\ of\ capital\ structure} = \frac{\$3.75\ million}{0.5} = \$7.5\ million$$

This $7.5 million level represents the next break point in the weighted (marginal) cost of capital schedule. This $7.5 million consists of: $3.75 million retained earnings, $0.75 million preferred stock and $3 million debt.

Beyond the second tranche, all additional funds raised will be with high cost debt, new common stock and preferred stock. The weighted cost of these funds is as follows:

$$WACC_3 = k(W_3) = 0.4 \cdot 6\% + 0.1 \cdot 10\% + 0.5 \cdot 15.9\% = 11.35\%.$$

III. Determining the optimal capital budget

The optimal capital budget can be determined by comparing the expected project returns to the company's marginal cost of capital schedule. We can illustrate this issue by drawing two curves: investment opportunity curve (IOS) and weighted (marginal) cost of capital curve (MCC). The optimal capital budget is indicated by the point at which the investment opportunity curve and the marginal cost of capital curve intersect.

Investment Opportunity Schedule (IOS) – a listing, or graph, of a company's investment opportunities ranked in order of a projects' rates of return.

Marginal Cost of Capital (MCC) schedule – a graph or table that relates a company's weighted average cost of capital to the amount of new capital raised.

TOR Company's optimal capital budget totals $23 million and includes projects A, B, C and D. Projects E and F are excluded, because their returns are expected to fall below 11.35% of cost of funding. Acceptance of projects E and F would result in a decrease in the company's value. In principle, the optimal capital budget maximises the value of a company.

It should be noted that this procedure assumes that all projects being considered are of equal (average) risk.

Capital Budgeting

Investment decisions affect the level and structure of assets, and the corresponding financial decisions affect the level and structure of liabilities and equity. These decisions are mutually independent in the sense that the attractiveness of equity investments does not depend on how they are financed. Hence, it can be concluded that the sources of obtaining funds necessary to launch investments do not affect the effectiveness of the projects under consideration, because the increase in funds is carried out in accordance with the capital structure theory.
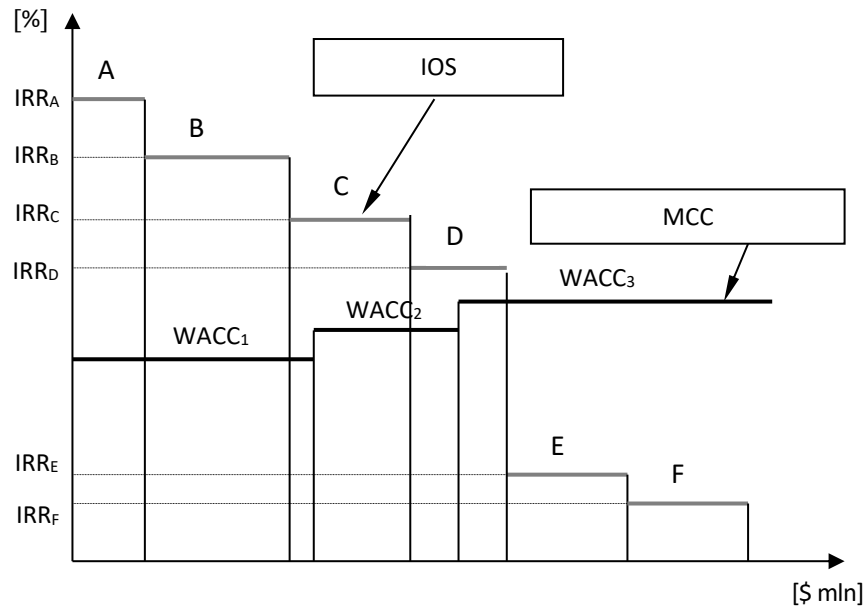

Figure 5.1. Determining the optimal capital budget

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